

# Modelling network performance with a spatial stochastic process algebra

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  - ▶ using a process algebra with stochastic, continuous and discrete aspects
- ▶ conclusions and further work



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  - ▶ points in  $n$ -dimensional space



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$$P ::= P \boxtimes_M P \mid P/M \mid C$$

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- ▶ other rules defined in the obvious manner
- ▶ instantiate functions to obtain concrete process algebra

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  - ▶ each sequential component must have single fixed location
  - ▶ communication must be pairwise and directional
- ▶ let  $\mathcal{P}_{\mathcal{L}} = \mathcal{L} \cup (\mathcal{L} \times \mathcal{L})$ , singletons and ordered pairs



# Functions for concrete process algebra

- ▶ functions



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## Functions for concrete process algebra

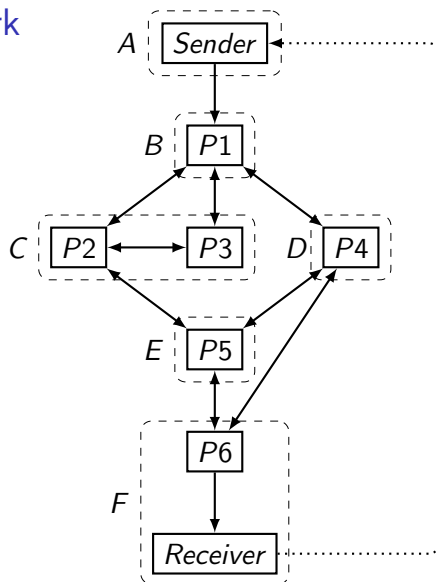
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$$rsync(P_1, P_2, L_1, L_2, r_1, r_2) = \begin{cases} \frac{r_1}{r_\alpha(P_1)} \frac{r_2}{r_\alpha(P_2)} \min(r_\alpha(P_1), r_\alpha(P_2)) \cdot w((\ell_1, \ell_2)) & \text{if } L_1 = \{\ell_1\}, L_2 = \{\ell_2\}, (\ell_1, \ell_2) \in E \\ \perp & \text{otherwise} \end{cases}$$

## Example network





## PEPA model

$$Sender@A \stackrel{def}{=} (prepare, \rho).Sending@A$$

$$Sending@A \stackrel{def}{=} \sum_{i=1}^6 (c_{Si}, r_S).(ack, r_{ack}).Sender@A$$

$$Receiver@F \stackrel{def}{=} \sum_{i=1}^6 (c_{iR}, r_R).Receiving@F$$

$$Receiving@F \stackrel{def}{=} (consume, \gamma).(ack, r_{ack}).Receiver@F$$

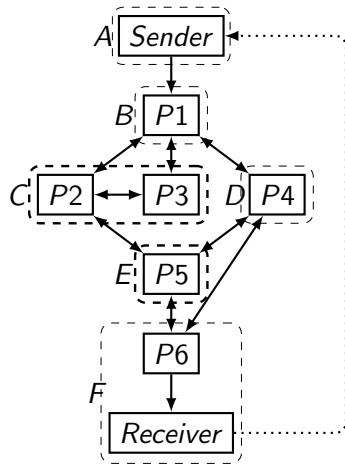
$$P_i@l_i \stackrel{def}{=} (c_{Si}, T).Q_i@l_i + \sum_{j=1, j \neq i}^6 (c_{ji}, r).Q_i@l_i$$

$$Q_i@l_i \stackrel{def}{=} (c_{iR}, T).P_i@l_i + \sum_{j=1, j \neq i}^6 (c_{ij}, r).P_i@l_i$$

$$Network \stackrel{def}{=} (Sender@A \underset{*}{\boxtimes} (P1@B \underset{*}{\boxtimes} (P2@C \underset{*}{\boxtimes} (P3@C \underset{*}{\boxtimes} (P4@D \underset{*}{\boxtimes} (P5@E \underset{*}{\boxtimes} (P6@F \underset{*}{\boxtimes} Receiver@F))))))))$$

# Graphs

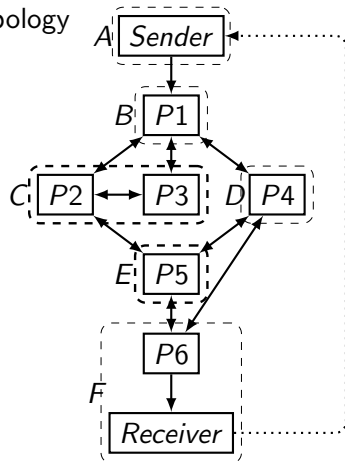
- ▶ rates:  $r = r_R = r_S = 10$



# Graphs

- ▶ rates:  $r = r_R = r_S = 10$
- ▶ the weighted graph  $G$  describes the topology

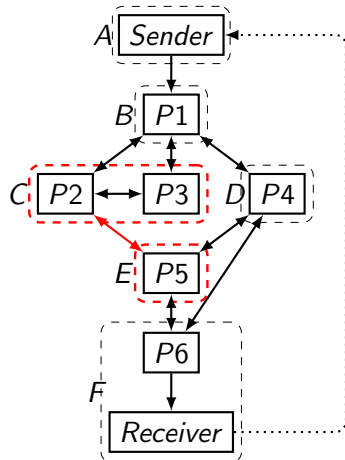
	A	B	C	D	E	F
A	1	1				
B			1	1		
C		1	1		1	
D		1			1	1
E			1	1		1
F	1			1	1	1



# Graphs

- ▶  $G_1$  represents heavy traffic between  $C$  and  $E$

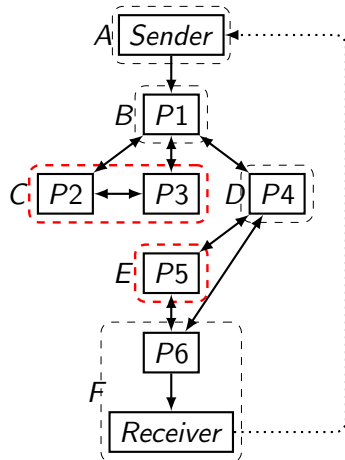
	A	B	C	D	E	F
A	1	1				
B			1	1		
C		1	1		0.1	
D		1			1	1
E			0.1	1		1
F	1			1	1	1



# Graphs

- ▶  $G_2$  represents no connectivity between  $C$  and  $E$

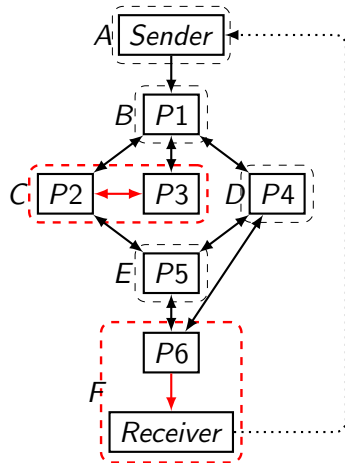
	A	B	C	D	E	F
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C		1	1		0	
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# Graphs

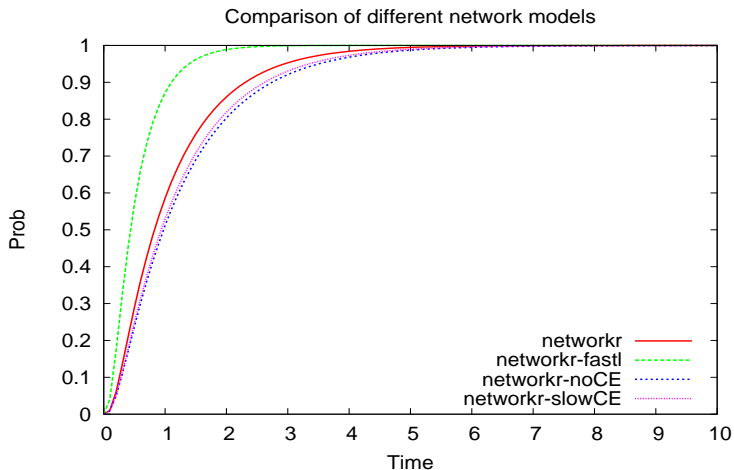
- ▶  $G_3$  represents high connectivity between colocated processes

	A	B	C	D	E	F
A	1	1				
B			1	1		
C		1	10		1	
D		1			1	1
E			1	1		1
F	1			1	1	10



# Analysis

- cumulative density function of passage time



# Evaluation

- ▶ uniform description for each node in the network



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- ▶ existing analysis framework
- ▶ abstract process algebra is flexible



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  - ▶ different types of networks
  - ▶ virus transmission in vineyards



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  - ▶ piecewise deterministic Markov processes
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  - ▶ periods of connectivity modelled stochastically
  - ▶ full buffers modelled discretely
  - ▶ determine storage required at nodes



# Conclusion and further work

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  - ▶ useful for modelling network performance
- ▶ further research
  - ▶ explore how it can be applied in modelling networks



# Conclusion and further work

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  - ▶ theoretical results for abstract process algebra
  - ▶ behavioural equivalences





Thank you

This research was funded by the EPSRC SIGNAL Project

## More comments

- ▶ related research



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  - ▶ PEPA nets (Gilmore *et al*)



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- ▶ locations and collections of location



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  - ▶ prove results for parametric process algebra

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  - ▶ prove results for parametric process algebra
  - ▶ then apply to concrete process algebra