

# MULTI-DWELLING REFURBISHMENT OPTIMIZATION: PROBLEM DECOMPOSITION, SOLUTION AND TRADE-OFF ANALYSIS

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## ABSTRACT

A methodology has been developed for the multi-objective optimization of the refurbishment of domestic building stock on a regional scale. The approach is based on the decomposition of the problem into two stages: first to find the energy-cost trade-off for individual houses, and then to apply it to multiple houses.

The approach has been applied to 759 dwellings using buildings data from a survey of the UK housing stock. The energy use of each building and their refurbished variants were simulated using EnergyPlus using automatically-generated input files.

The variation in the contributing refurbishment options from least to highest cost along the Pareto front shows loft and cavity wall insulation to be optimal initially, and solid wall insulation and double glazing appearing later.

## INTRODUCTION

Housing is responsible for more than a quarter of total energy consumption and carbon dioxide (CO<sub>2</sub>) emissions the UK (DECC, 2014). With less than 1% annual growth rate of new-build homes, it is estimated that 75% of the housing stock in 2050 will have been constructed before 2014 (Ravetz, 2008). Although energy efficiency for the whole housing stock has increased slightly over the years, the average home energy rating remains low and the housing stock could hugely benefit from a wide range of retrofit measures (DCLG, 2014). In order to achieve the UK Government's CO<sub>2</sub> reduction target of 80% by 2050 compared to the 1990 baseline (HM Government, 2008), large-scale retrofitting (i.e. improving the thermal efficiency and energy system efficiency of dwellings rapidly and at high volumes) of the existing housing stock is expected to play an important role.

In general, available retrofit measures can be categorized into three groups:

- Improving the building envelope, e.g. insulating walls, roofs, and windows;
- Improving heating and hot water systems, e.g. upgrading boilers and control systems;

- Installing renewable energy systems, e.g. photovoltaic, biomass boilers, ground source heat pump systems, etc.

When retrofitting a building, there is usually more than one measure that is applicable and the capital cost and energy saving of one measure could be very different from that of another. Consequently, it is of great importance to identify the most cost effective combinations of retrofit measures. At a regional or national level, the identification of which retrofit strategies should be targeted for government incentives is particularly difficult as at these scales, the most cost effective strategy is influenced by: the large number of dwellings that could be refurbished; the high number of alternative forms of dwelling; and the existing level of refurbishment of each dwelling, this influencing the extent to which further refurbishment is possible.

Applying multi-objective optimization to evaluate the retrofit strategies for a single building by considering multiple and competing objectives, such as cost, energy saving and thermal comfort, is well established in the research field of building simulation (Asadi et al., 2014; Evins, 2013; Wright et al., 2002). In contrast, applying multi-objective optimization to a large scale retrofitting program on multiple buildings, particularly on a regional or national scale, is still emerging. Scaling the optimization problem up from a single building to many buildings results in a very large search space and objective functions that are insensitive to any single instance of refurbishment; for instance, the reduction in heating energy use resulting from the installation of wall insulation in a single dwelling will be very small in relation to the total heating energy use of the region. The large scale and the insensitivity of the objectives to the variables results in a problem that is difficult to solve and that requires a very large number of candidate building performance simulations.

This paper describes the development of an optimization methodology for the multi-objective optimization of domestic building stock on a regional scale, that is computationally efficient, and which has proved to be robust in finding the trade-off between the reduction in building energy use and capital cost for multiple dwellings. The optimization approach is

based on the decomposition of the problem into two stages, the first being to find the energy-cost trade-off for individual houses, and the second for the trade-off of multiple houses at a regional scale.

## LARGE SCALE OPTIMIZATION

There has been recent interest among the evolutionary computation community in large scale optimisation, with a recent review (Latorre et al., 2014) citing a number of different approaches, and the introduction of the CEC large-scale benchmark problems (Li et al., 2013). Much of the literature focuses on problems with around 1000 variables, although some (Deb et al., 2003; Sastry et al., 2007; Semet and Schoenauer, 2005) do tackle very large problems with millions or more variables. The specific application in the present paper has the equivalent of 1205 binary variables (and has the potential for solving problems with more variables). Typically, approaches for the very large problems use hybrid algorithms that exploit characteristics of the application such as known partial solutions and constraints to reduce the search space and improve efficiency (Deb et al., 2003; Semet and Schoenauer, 2005). There is some evidence (Durillo et al., 2010, 2008) that commonly-used evolutionary multi-objective optimization methods do not scale well with the number of decision variables, and as far as the authors are aware, there have been few attempts so far (an exception being a study by Antonio and Coello Coello (2013)) at multi-objective optimization of problems with thousands of decision variables. This motivates the development of frameworks for large scale multi-objective problems.

## METHODS

### English housing survey (EHS) data

The English Housing Survey (EHS) is a year-on-year national survey commissioned by the UK Department for Communities and Local Government (DCLG). It collects information about people's housing circumstances and the condition and energy efficiency of housing in England (DCLG, 2015). Its database provides detailed information, such as age band, dwelling type, region, dimensions, window area, glazing type, wall construction, roof construction, floor construction, loft insulation and built form, of representative houses in England. This detailed information can be interpreted to allow the performance of the recorded dwellings to be simulated dynamically in building simulation software such as EnergyPlus (D.B. et al., 2000).

This study is focused on regional planning for the NE of England. The 2009 EHS database contains 935 sample dwellings in the NE region of England, and these dwellings represent 1.2 million homes in that region. The 935 dwellings are of 6 different dwelling types, 10 age bands, 8 wall construction types, and 12 loft insulation levels. The distribution of the 935 dwellings in dwelling type, age band, wall

construction and loft insulation is shown in Figure 1. This initial study excludes flats and focuses on the 759 houses recorded in the database, taking no account at this stage of the 1.2 million homes they represent. Note that although the selected stock relate to 4 high-level archetype forms (detached, semi-detached, mid-terrace, and end-terrace), the robust identification of the refurbishment strategy for the region requires the variants of each archetype to be included in the optimization (the variants relating particularly to the type of construction and state of refurbishment of each dwelling).

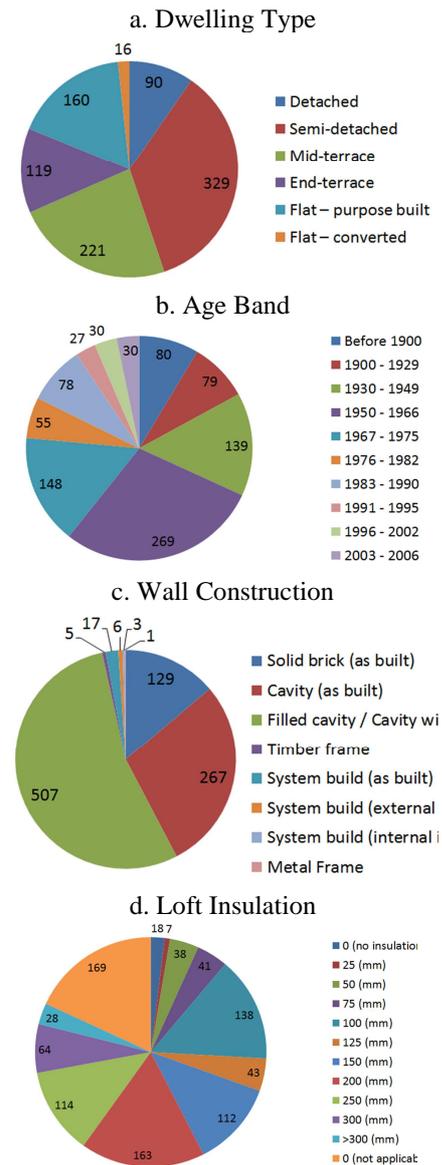


Figure 1. Distributions of 935 dwellings in dwelling type, age band, wall construction and loft insulation.

### Energy demand of the housing stock

A dynamic housing stock model has been developed, using the English Housing Survey (EHS) 2009 data, coupled to the EnergyPlus dynamic simulation

engine. EnergyPlus is a well-recognized and extensively tested fully-integrated building simulation tool and freely available. EnergyPlus takes an input data file (IDF), in which a building model is specified, and a weather file to run a dynamic simulation of a building. Although there are tools currently available to create IDFs, none of them is suitable to simulate a relatively large number of real houses with individual dimensions, different age bands and various fabric constructions. Therefore, an in-house program called the Building Generation Tool (BGT) has been developed to create the IDFs automatically, taking inputs from text files. The detailed description of the model and the validation against a steady-state housing stock model can be found in a previous study (He et al., 2014).

In order to minimize computational complexity, the approach adopted for simulating the thermal behavior of the houses was to determine the heating demand. Post-processing was then used to account for fuels and heating systems. The optimization study was therefore restricted to retrofit measures related to improving the building fabric, including cavity wall insulation, internal solid wall insulation, external solid wall insulation, loft insulation, and double glazing. All possible individual or combined retrofit measures have been applied to each of the 759 houses identified in the EHS 2009 database. However, not all measures are applicable to all houses; for example, double glazing is not an option for houses that are already fully double glazed. This reduced the number of possibilities with only 2097 retrofit strategies being applicable across all 759 houses. The energy demand of the original 759 houses and the retrofitted 2097 houses has been estimated by the dynamic housing stock model (He et al., 2014).

### Costs of retrofit measures

Despite there being multiple price guide books and references available, no single source of cost information could be found that covers all retrofit measures and the range of figures varies widely from different sources (Porritt, 2012). Table 1 shows the costs of retrofit measures used in this study.

These costs were carefully chosen to reflect the real costs in the market. Most of the costs are taken from Energy Saving Trust (EST, 2015), except the cost of double glazing which is taken from a report from the Retrofit for the Future project by the Technology Strategy Board (TSB, 2014).

### Multi-objective optimization

A multi-objective optimization package has been applied to identify the most cost-effective combinations of all measures across the housing stock. The algorithm selected in this study is an implementation of the popular Non-dominated Sorting Genetic Algorithm II (NSGA-II) first proposed by Deb et al. (2002). The same

implementation was used in a previous study to optimize the design of fenestration on a façade of a building (Wright et al., 2013).

Table 1 Costs of retrofit measures.

RETROFIT OPTIONS	DWELLING TYPE	COST	SOURCE
Loft Insulation (0 to 270mm)	Detached	£395	(EST, 2015)
	Semi/End	£300	
	Mid	£285	
Loft Insulation top up (100mm to 270mm)	Detached	£265	(EST, 2015)
	Semi/End	£220	
	Mid	£215	
Cavity Wall Insulation	Detached	£720	(EST, 2015)
	Semi/End	£475	
	Mid	£370	
Solid Wall Internal Insulation	External wall area	£87/m <sup>2</sup>	(EST, 2015)
		£157/m <sup>2</sup>	
Solid Wall External Insulation	External wall area	£157/m <sup>2</sup>	
Double Glazing	Window area	£261/m <sup>2</sup>	(TSB, 2014)

Two objectives are optimized: the energy demand of the housing stock, estimated by the dynamic housing stock model; and the costs of installing retrofit measures, calculated using the values in Table 1. In order to deal with the large search space, the optimization approach is decomposed into two stages. First, an exhaustive search is run for each individual house to find its Pareto-optimal energy-cost trade-off (the search space for this typically being a maximum of a few tens of solutions per house). Although it was not done in the present work, it is possible at this point to take constraints into account such as over-heating risk, so that only admissible solutions are passed to the next stage. Secondly, to find the trade-off for the housing stock, the space of Pareto-optimal solutions over all houses is searched using NSGA-II. Each house is represented in the second stage problem by a single integer variable whose value identifies the specific Pareto-optimal solution for the house arising from the first stage. The Pareto-optimal solutions for each house are sorted by cost, so that there is a natural ordering over the values that the variable for a house can take. A parametric tool called jEPlus (Zhang, 2009) has been used in this study to run simulations in EnergyPlus in parallel and to extract outputs. Each simulation takes about 30 seconds to run; therefore running a full set of simulations for the original 759 houses and the potential retrofitted 2097 houses takes approximately 6 hours in a dual-core PC with 4 threads.

### The optimization approach explained

As noted earlier, the two stage optimization was developed to improve search efficiency over the large search space. Considering the possible refurbishment

options that can be applied to each of the 759 houses in the stock, the overall problem has a total of 1205 decisions about whether to apply a specific refurbishment. The resulting problem has a search space of  $2^{1205}$  (approx.  $5.5 \times 10^{362}$ ) solutions. Using the two stage approach exploits the fact that the houses are independent. While the houses are related in the sense that a decision to spend money refurbishing one house means that it cannot be spent on a different one, the more complex relationship between energy saving and refurbishment cost is independent for each house (unless there is, for example, a community shared heating system, but this is not considered here). Separated into two stages, the search space at each stage is reduced. At stage 1 (exhaustive search across all houses), rather than the product of all possible options across the whole stock, we only consider the product of possible solutions for each house separately. For any one house, there are a maximum of 5 refurbishment options, so the search space for each house has a maximum of 32 solutions. At stage 2, only the product of Pareto-optimal solutions found at Stage 1 are considered, resulting in a reduced search space over 759 variables (1 per house) of  $3.1 \times 10^{300}$ . This is still very large but represents a substantial reduction from the overall problem. Moreover, the second stage optimization is on solutions that are known to be Pareto optimal and therefore, the limits of the search space are naturally constrained to be in the region of the optima (which may improve the effectiveness of the second stage search).

To explore the alternative optimization approaches, a synthetic example problem was developed: a version of the stock optimization problem using the same house repeated 500 times. Using duplicate houses allows a reference Pareto front to be developed by taking each of the solutions from the house's front and scaling up their energy and cost by a factor of 500.

The true Pareto front for the stock will not perfectly match this reference front (it will almost certainly have many more solutions, some of which may dominate those in the reference front) but it provides a useful approximation to work towards. By way of illustration, consider a single house with three Pareto-optimal solutions: (1,5), (3,4), (5,1) where the x and y values might represent cost and energy consumption. The "Reference Pareto front" for a stock of three houses, are the points (3,15), (9,12), (15,3). However, at the stock level, different refurbishments might be applied to each of the three houses: there are 27 possible combinations of the Pareto-optimal solutions for each house, some of which have identical values, plotted as "All" Figure 2. The "Global Pareto front" of these lies near to the reference front, but does not include the point at (9,12).

For this house, there were 10 refurbishment options, giving a search space of  $2^{10}=1024$  per house, and

$2^{10 \times 500} = 1.4 \times 10^{1505}$  overall. This was deliberately larger than the real problem studied in this paper, to ensure that the approach would scale to larger problems. An exhaustive search was conducted over the 1024 solutions for a single house, these options being plotted in terms of the house's resulting annual operational energy demand and capital cost of the refurbishments in Figure 3. The Pareto-optimal solutions from this search are highlighted.

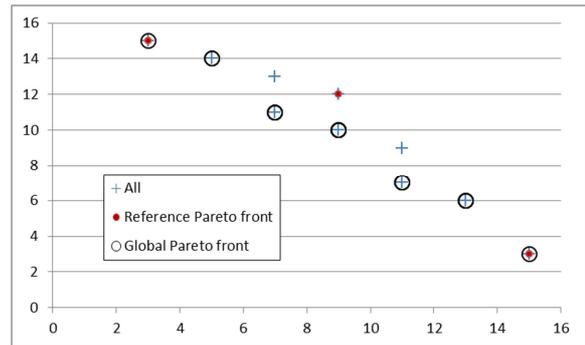


Figure 2. Combining solutions from the Pareto fronts for individual houses into a global Pareto front does not simply result in a scaled up version of the front for one house.

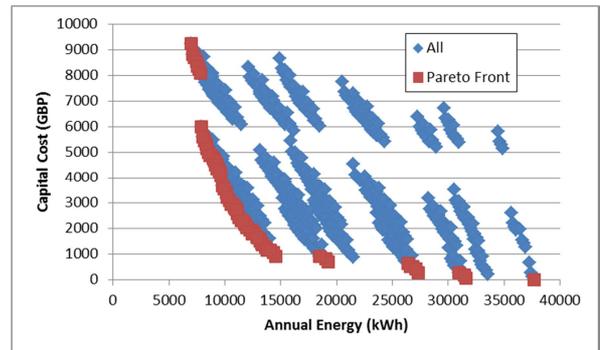


Figure 3. Exhaustive set of refurbishment options for one house in the synthetic example problem, with the Pareto optimal solutions highlighted.

The example problem was used to test several approaches to the multi-objective optimization using NSGA-II. A naïve approach to tackling the global problem would be a binary encoding, in which each bit represents a possible refurbishment on a particular house: 2000 bits for the example. Secondly, the two-level approach could be taken, using either a Gray binary or integer encoding. All algorithm runs were terminated after 100,000 evaluations, with a population size of 200 and binary tournament selection. Those with a binary encoding used bit flip mutation at a rate of 1/2000, and uniform crossover at a rate of 1.0. The integer encoded algorithm used simulated binary crossover and polynomial real mutation. The results for each are plotted against the reference in Figure 4. These are from single runs so

the individual solutions in the Pareto fronts can be shown, but the same broad trend was reflected over multiple runs.

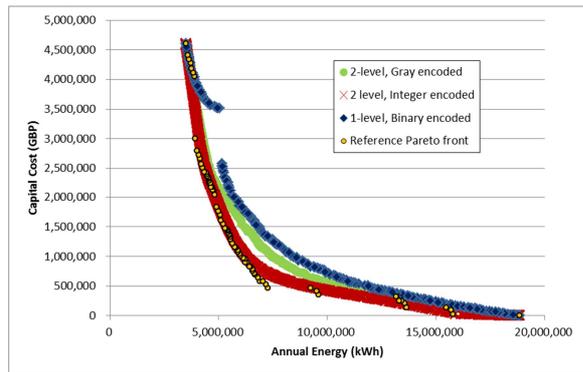


Figure 4. The global Pareto optimal fronts found by three different approaches to the synthetic example problem, compared with the reference front formed by scaling up the front for one house.

The integer-encoded 2-stage optimization finds a front that is closer to the reference front than the other two approaches. This is because it has a smaller search space to explore, and because the integer encoded algorithm was able to more efficiently explore the fronts for each house. It was found by experimentation that the approaches could be improved further by seeding the evolutionary algorithm with points from the reference front – that is, including these among the randomly generated solutions at the start of the algorithm’s run. However, it should be noted that this would not be an option for the real stock optimization problem, as the houses are not duplicates, making it impossible to generate a reference front by simply scaling up the front for one house.

## RESULTS AND DISCUSSION

### Pareto optimal result

The output set of non-dominated solutions, i.e. the Pareto optimal set, was derived from the set of all solutions generated over an optimization run. The parameters, such as random initialization number, which might affect the results of the optimization runs, have been extensively tested. The run found 398 solutions in the trade-off, which are plotted in the objective space, i.e. energy demand and cost, in Figure 5. The total heating energy demand of the 759 houses without applying any retrofit measures is about 5,660 MWh. Providing a total investment of £250,000, a maximum 310 MWh reduction in total heating energy demand (5,350 MWh) can be achieved through applying the optimum combination of retrofit measures to the houses. The cost-effective ratio of the investment can be defined as:  $R=ER/C$ , where R is the cost-effective ratio (kWh/£), ER is the total energy reduction (kWh) and C is the total cost (£). The cost-effective ratio of the initial £250,000 investment is 1.24 kWh/£. Increasing the investment

to £750,000, a maximum further 230 MWh reduction can be made, which gives a cost-effective ratio of 0.46 kWh/£ for the additional £500,000 investment. If all suitable measures are applied to all houses, it will cost a total of £1,620,000, reducing the total energy demand to 5,015 MWh. The further additional investment of £870,000 gives a cost-effective ratio of only 0.12 kWh/£.

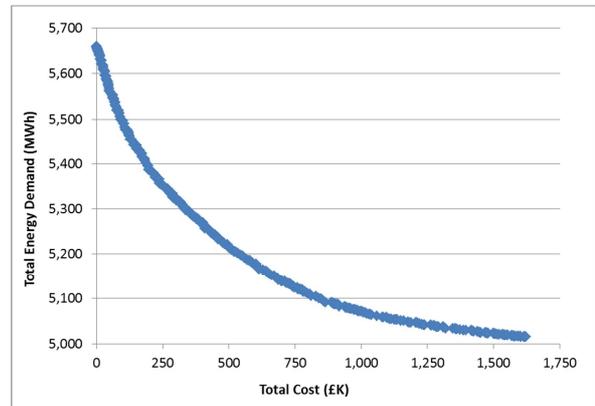


Figure 5. The Pareto optimal set found by the optimization, plotted in objective space.

### Data analysis

Analyzing the Pareto optimal solutions in detail provides insights into the uptakes of individual or combined retrofit measures during the optimization process. Figure 6 shows the number of installations for each individual retrofit measure for all the solutions on the Pareto front.

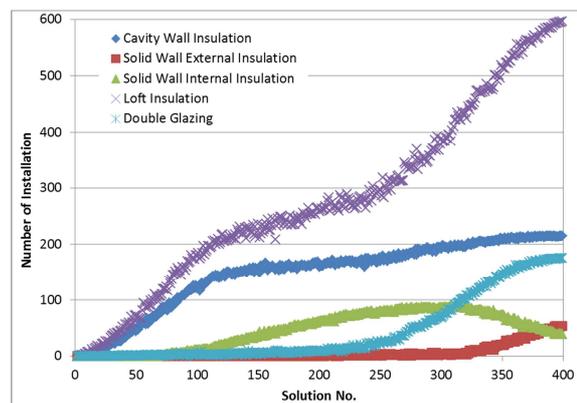


Figure 6 Counts of individual measures for the Pareto solutions.

The solutions on the Pareto front are ranked from 1 to 398 according to the increment of total cost, and this ranking is shown on the x-axis. Solution No.1, for example, is the solution with the minimum cost (0 in this case), and none of the measures is installed. Solution No. 398 is the solution with the maximum cost, and in this case, all the suitable measures for all houses are installed. Each line between Solution No.1 and No.398 shows the trend of the installation of a

particular measure as the total cost increases. The prioritization is in the installation of loft insulation, followed by the installation of cavity wall insulation, which is not surprising, considering that loft insulation is the cheapest, and cavity wall insulation second cheapest, among all selected measures and their energy savings are relatively high. External and internal solid wall insulation are two exclusive measures, both of which can only be applied to houses with solid wall construction. Internal insulation increases up to Solution No. 325 due to its lower cost; however, external insulation has a better performance in terms of reducing heat demand in some cases, and therefore starts to pick up at the higher cost end of the solutions, as internal starts to decline. The installation of double glazing also only starts to happen towards the high cost end of the solutions due to the high cost and the smaller savings from individual installation.

## CONCLUSIONS

A two stage optimization methodology has been proposed for approximating the trade-off between the reduction in building energy use and capital cost for refurbishments of the domestic building stock on a regional scale. The methodology has been shown to work well for a synthetic problem covering 500 duplicate houses each with 10 possible refurbishment options. It was then successfully applied to the multi-objective optimization of refurbishments to 935 sample dwellings in the NE region of England. These were taken from the 2009 EHS database and represent 1.2 million homes.

When resources are limited, as is often the case in the real world and particularly in a large scale retrofitting programme, it is important to identify the most cost-effective measures that can be applied to the most suitable houses. By applying the multi-objective optimization package, it is possible to derive the Pareto optimal solution set that demonstrates the trade-off between the energy demand and cost. The findings show that the cost-effective ratio decreases sharply for a significant increase in investment. The initial £250,000 investment could result in a cost-effective ratio of 1.24 kWh/£, while the cost-effective ratio of an additional £500,000 investment drops to 0.46 kWh/£. Retrofitting all houses with the expensive measures for a further additional £870,000 causes the cost-effective ratio to fall to 0.12 kWh/£.

The analysis of the Pareto optimal solutions set can be complicated, particularly where there is a large number of an individual or combined retrofit option to consider. A simple approach based on ranking the solutions and counting the number of installations of individual measures, whether they are applied on their own or in combination with other measures, has been used in this study to examine the trend of installation for each measure across the whole cost range. While it is not surprising the uptake of loft insulation shows a much faster trend, followed by

cavity wall insulation, it is interesting to notice the uptake of double glazing only begins towards the higher cost end, and the uptake of solid wall internal insulation starts to drop and later overtaken by solid wall external insulations at the higher cost end.

## FURTHER WORK

When running all the dynamic simulations, the overheating hours for living room and bedroom were recorded, and in future work they will be added as the constraints in the optimization process. Furthermore, the ability to predict dynamic demand at regional or sub-regional level by the dynamic housing stock model needs to be further investigated. The optimization methodology has worked for the synthetic optimization problem, which has a larger number of variables (5000) than the problem derived from the real North-East housing stock data (1205). With improvements to the modelling to cover more housing types and more refurbishment options, the problem size will grow and we intend to apply the framework to these larger scale problems. Preliminary studies suggest that it can be scaled to a problem with over 16,000 houses (with nearly 50,000 variables). As well as improvements to the optimization algorithm itself, more advanced methods to analyse the resulting Pareto-optimal fronts will need to be developed to make sense of the large amount of information arising from such an optimization.

## ACKNOWLEDGEMENT

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