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A Temporal Model and Distance Metrics for Network Analysis

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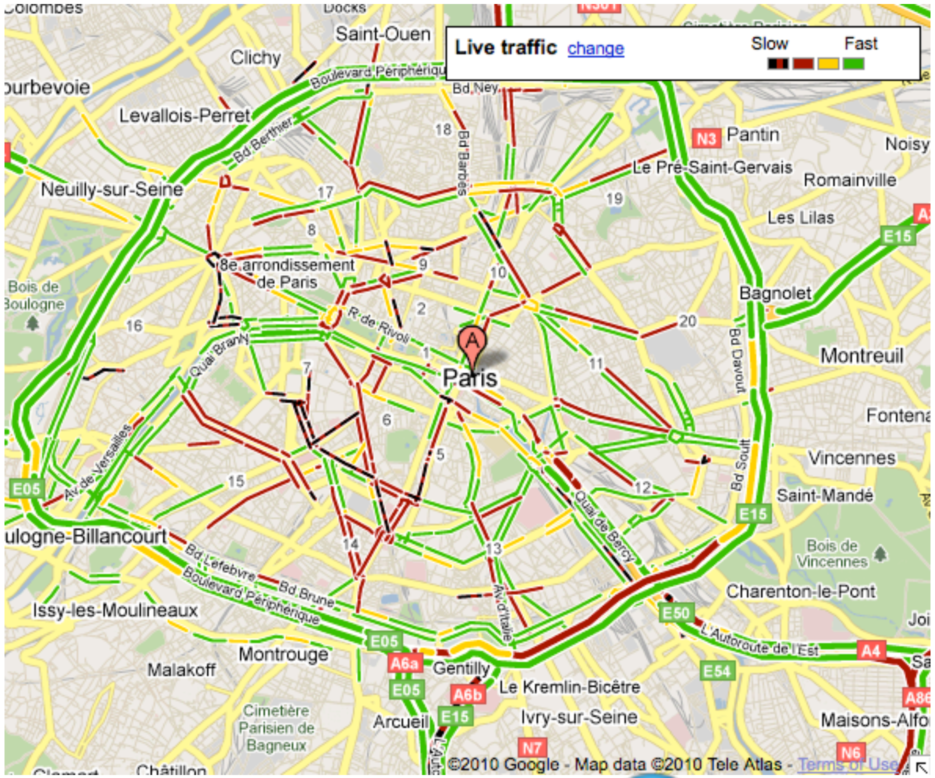
17th June 2010, University of Stirling

Some Real Networks



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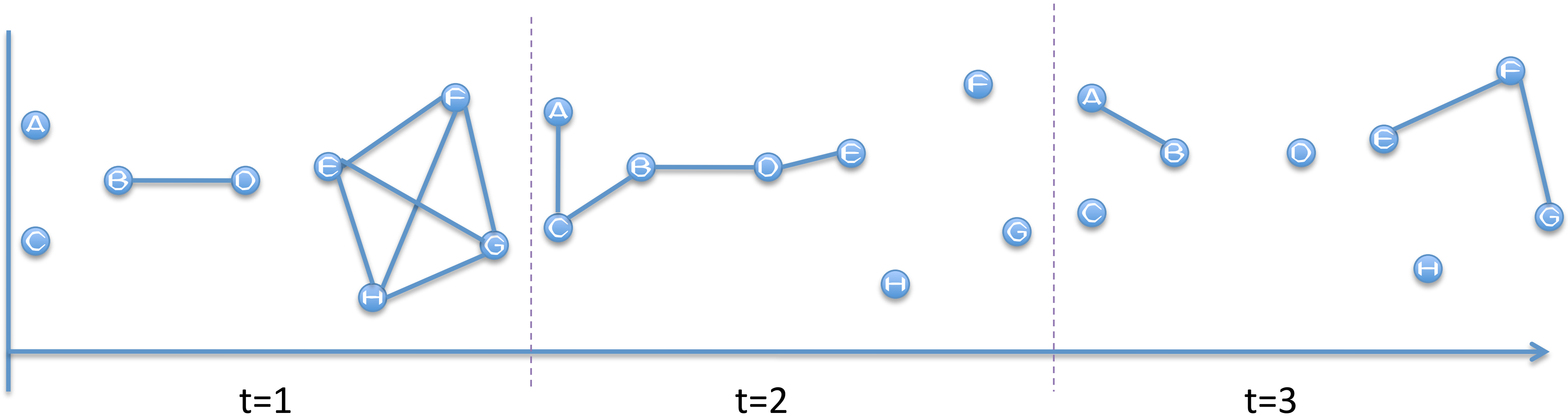
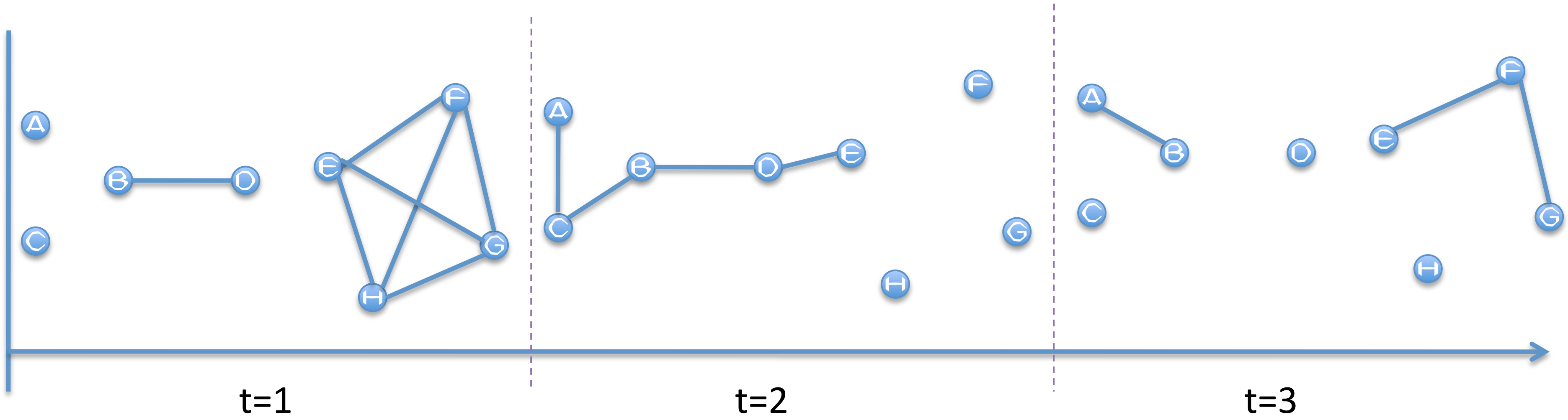
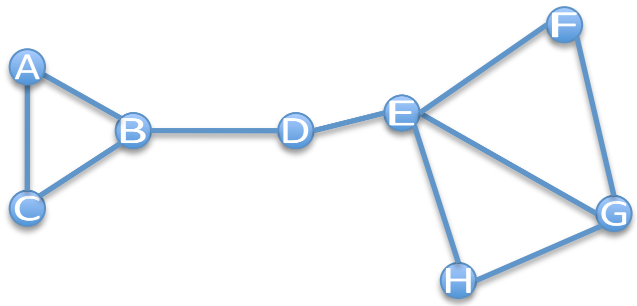
Temporal Graph

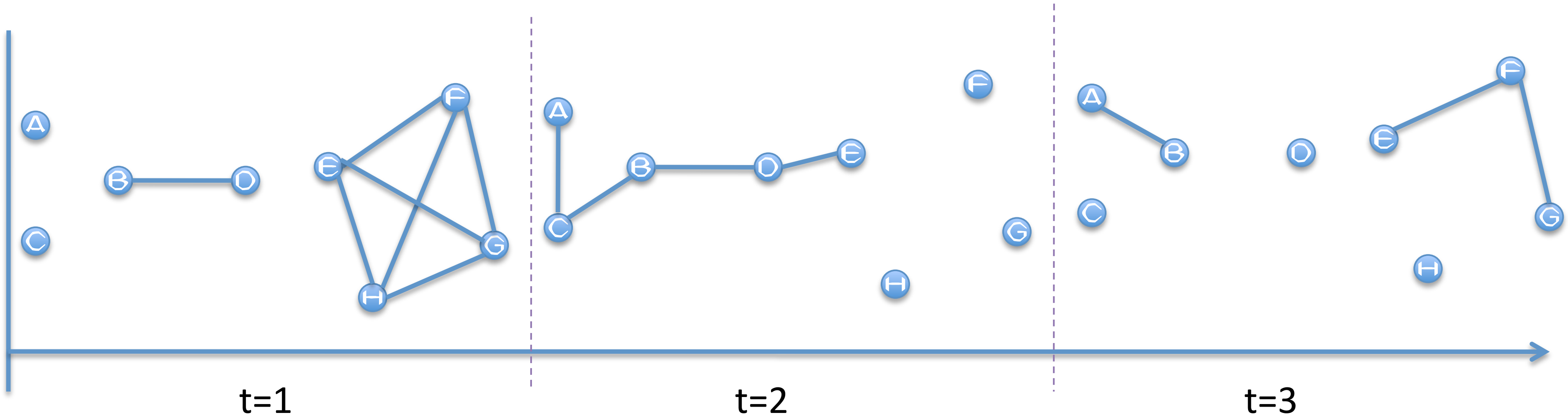


Temporal Graph



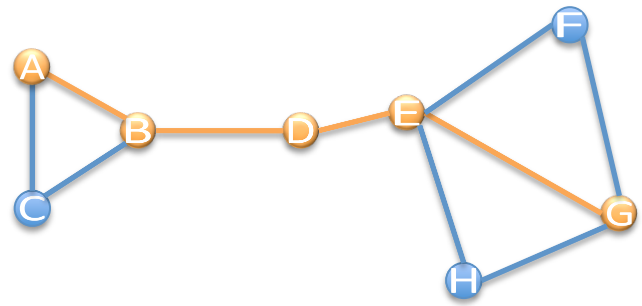


Temporal Graph



•Static

- •Shortest path (A,G) = [A,B,D,E,G]
- •Shortest path length (A,G) = 4 hops



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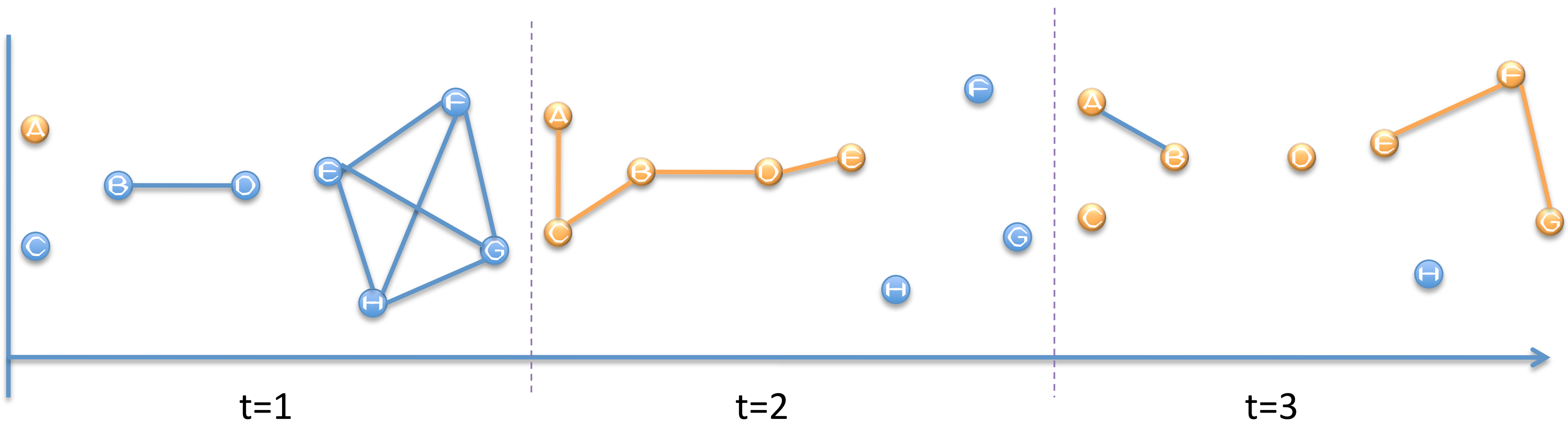
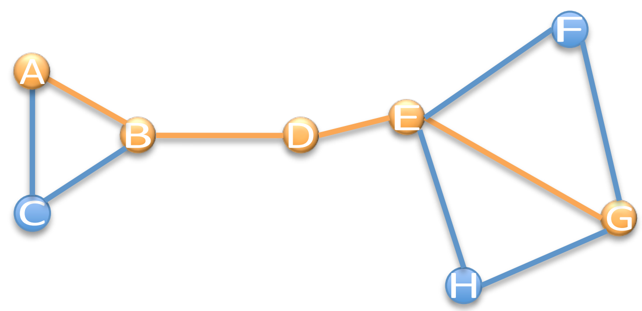
Temporal Graph



- Static
    - Shortest path (A,G) = [A,B,D,E,G]
    - Shortest path length (A,G) = 4 hops
- Temporal
    - Shortest path (A,G) = [A,C,B,D,E,F,G]
    - Shortest path length (A,G) = 6 hops
    - Time=3 seconds



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Temporal Metrics

- d_{ij} Shortest Temporal Path Length
- d^*_{ij} Shortest Path with temporal constraints
- $E_{ij} = \frac{1}{d_{ij}}$ Temporal Efficiency

Temporal Metrics

- Average Temporal $L = \frac{1}{N(N-1)} \sum_{ij} d_{ij}$

- Average Temporal $L^* = \frac{1}{N(N-1)} \sum_{ij} d^*_{ij}$

- Average Efficiency $E_{glob} = \frac{1}{N(N-1)} \sum_{ij} E_{ij}$

Does it really matter?

- Infocom 2005 conference environment
- Bluetooth colocation scans
- 5 Minute Windows
- Measure 24 hours starting 12am

					Static		Temporal		
Day	N	<k>	Activity	Contacts	L	Eglob	L*	L	Eglob
1	37	25.73	6pm-12pm	3668	1.291	0.856	4.090	19h 39m	0.003
2	39	28.31	12am-12pm	8357	1.269	0.870	4.556	9h 6m	0.024
3	38	22.32	12am-12pm	4217	1.420	0.798	4.003	10h 32m	0.018
4	39	21.44	12am-5pm	3024	1.444	0.781	4.705	9h 55m	0.013

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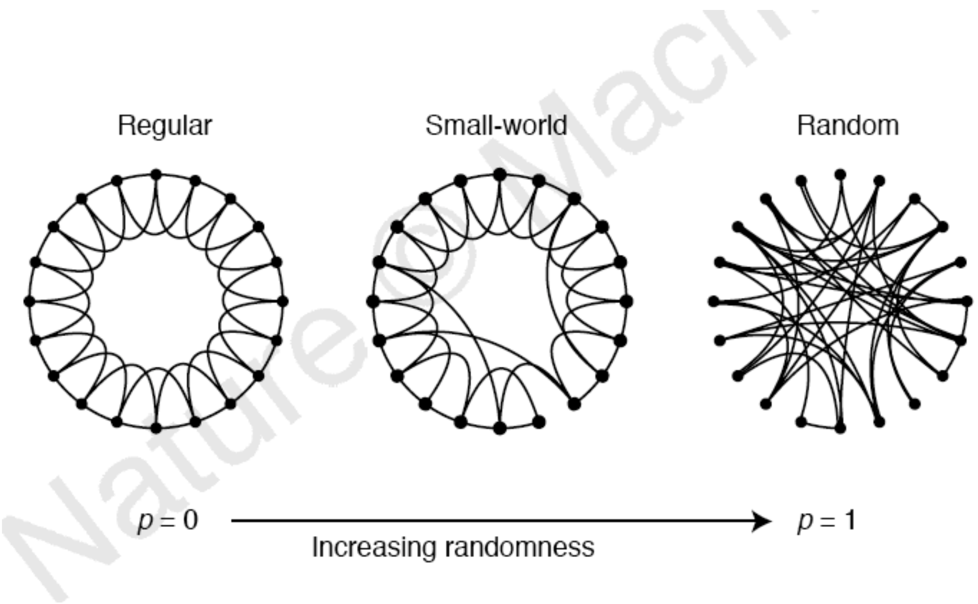
Temporal Small World

- Investigate speed of evolution of temporal graphs vs. communication efficiency
- *Intuition*: Slowly evolving graphs should be **slow** for data communication

Static SW Model

- Static
  - High local clustering
  - Some nodes provide short cut links



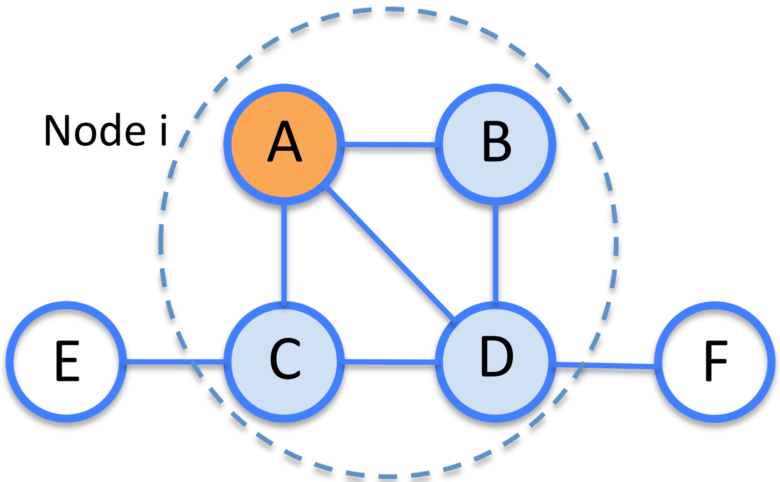
[Watts&Strogatz 1998]

Static Clustering Coefficient

$$C = \frac{\sum_i C_i}{N} \qquad C_i = \frac{2\sum_{j,k} a_{jk}}{[(\sum_j a_{ij}) * ((\sum_j a_{ij}) - 1)]}$$

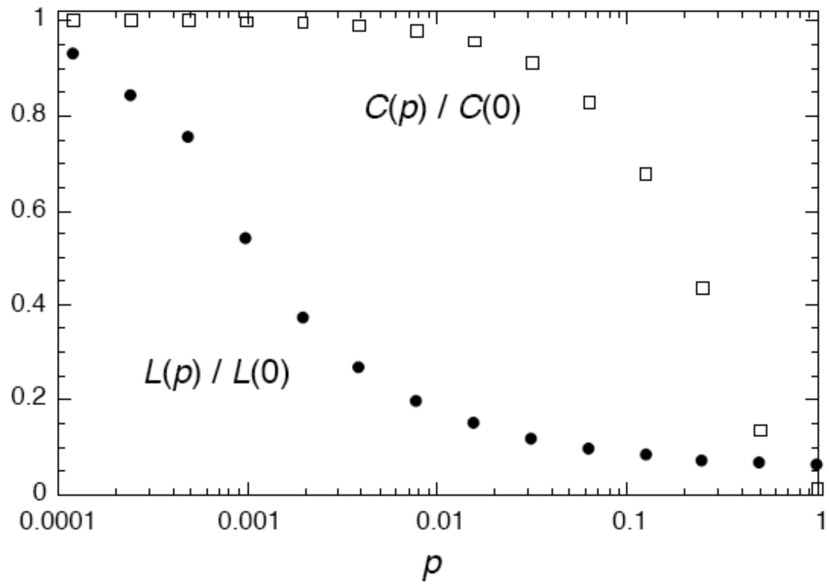
For all j, k such as $a_{i,j} = 1$ and $a_{j,k} = 1$

$$C_A = 2/3$$



Static Small World

- Graphs which both are locally clustered but with small average delay
  - High local clustering => Lattice
  - Small average delay => Random



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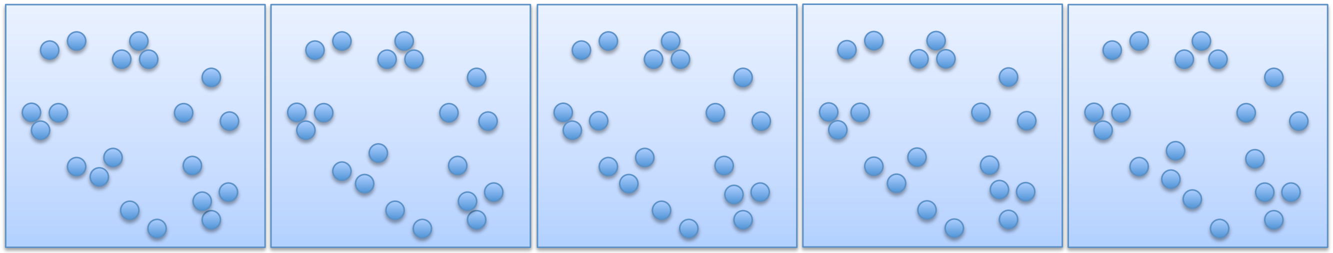
Temporal SW Model

- N Random Walkers with Prob Jumping P_j

P_j=0.0



Temporal SW Model

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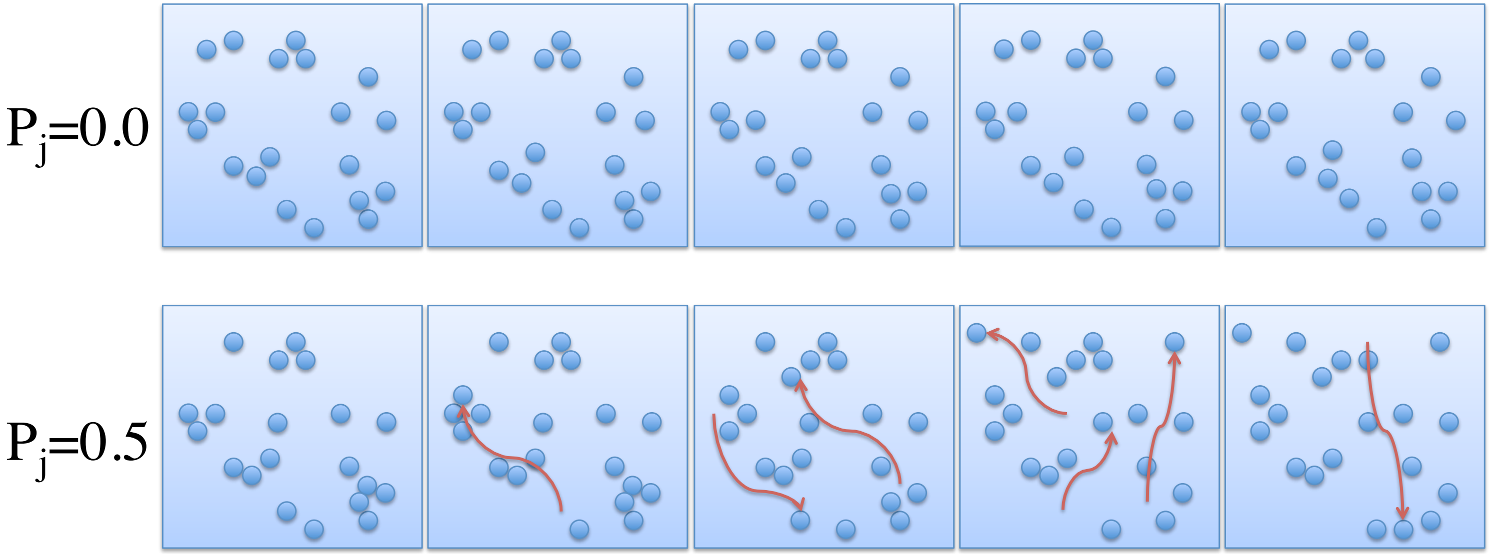

Temporal SW Model

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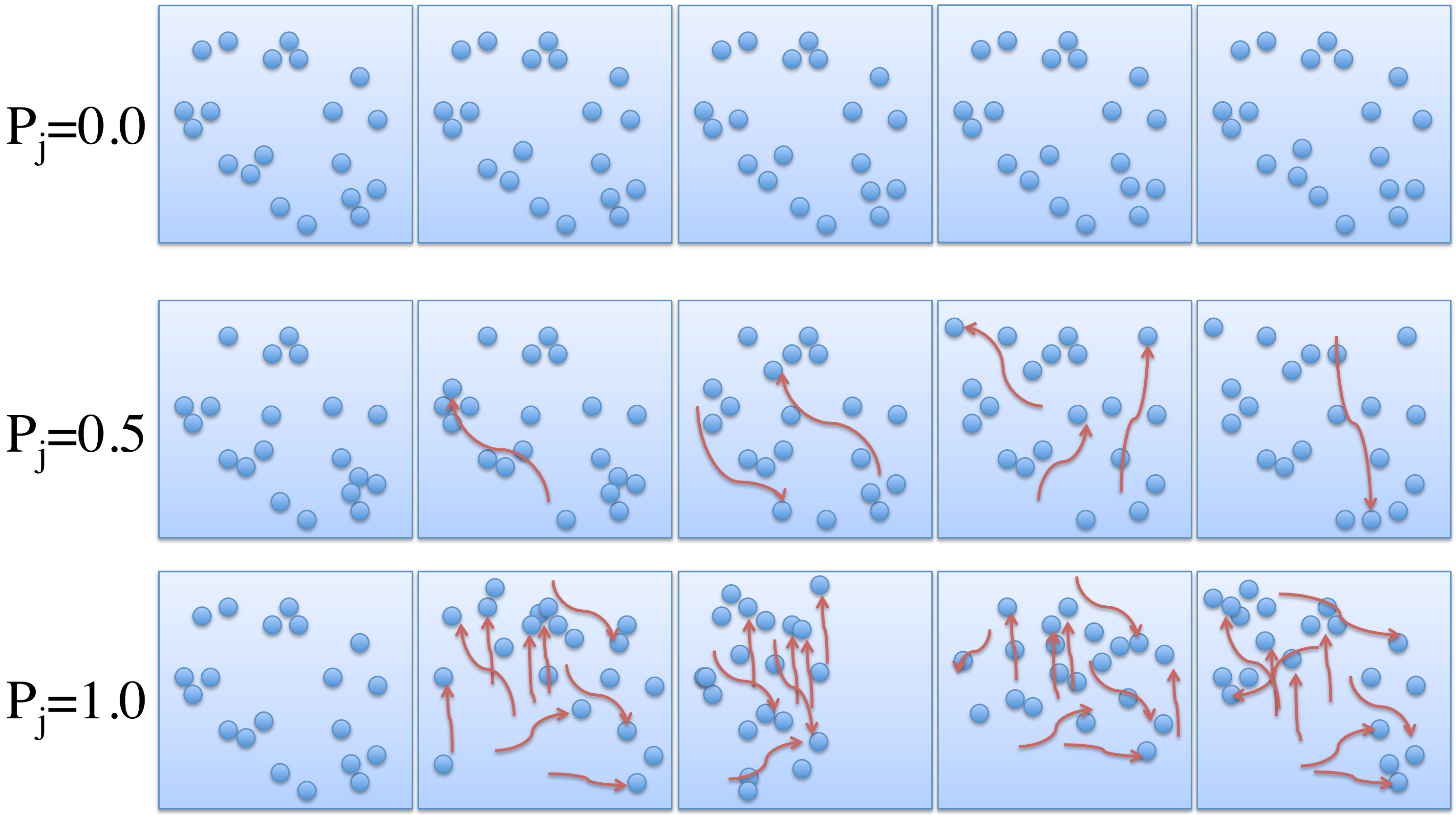

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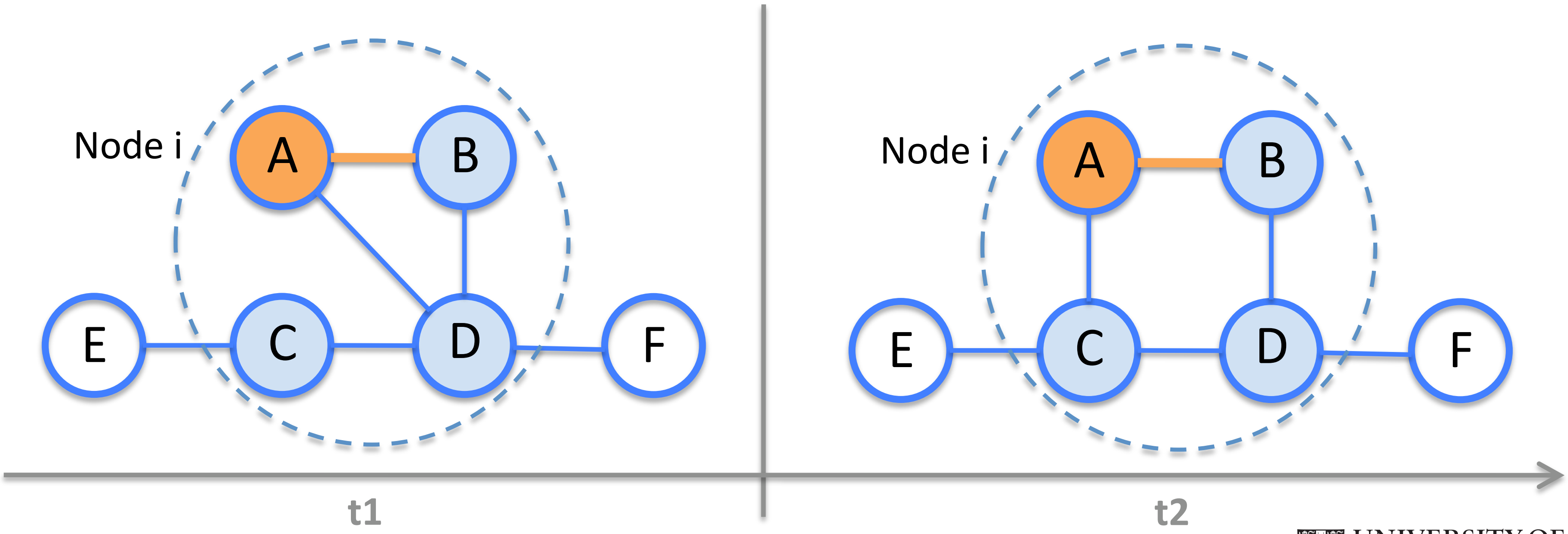
Temporal Correlation Coefficient

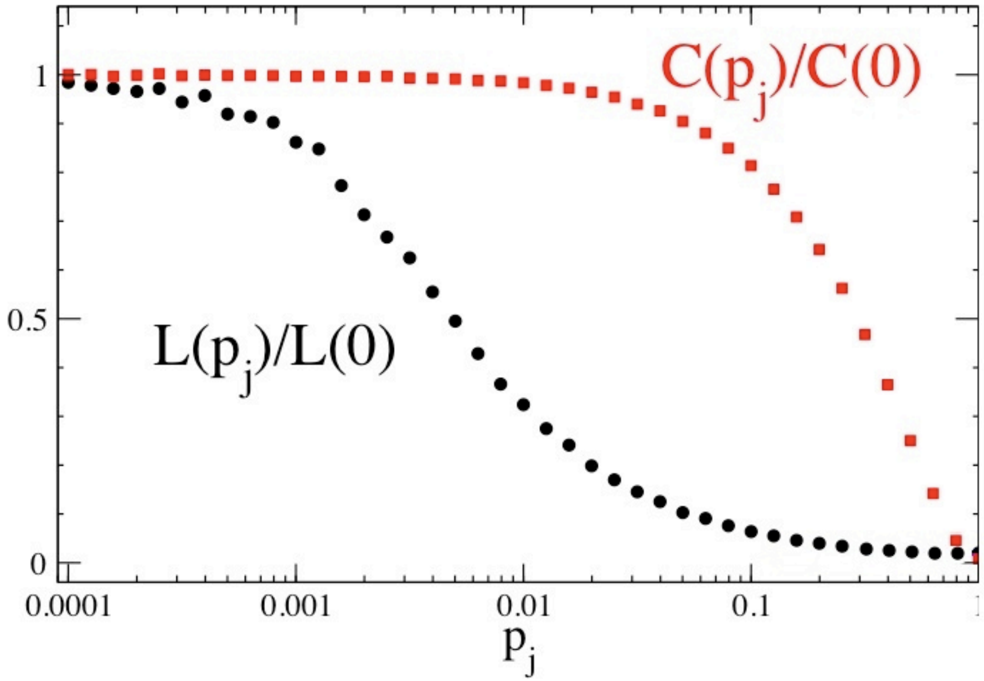
$$C = \frac{\sum_i C_i}{N} \quad C_i = \frac{1}{T-1} \sum_{t=1}^{T-1} \frac{\sum_j a_{ij}(t) a_{ij}(t+1)}{\sqrt{[\sum_j a_{ij}(t)][\sum_j a_{ij}(t+1)]}}$$

$$C_A = 1/2$$

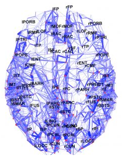


Temporal Small World

- Graphs which evolve slowly over time can still exhibit high communication efficiency
  - Highly temporal-clustering => non-jumping model
  - Low temporal-delay => fully-jumping model



Small-world Behaviour in Real Data



Brain network

Bluetooth contacts
(INFOCOM'06)

facebook
(London network)

	C	C^{rand}	L	L^{rand}	E	E^{rand}
α	0.44	0.18	3.9 (100%)	4.2 (98%)	0.50	0.48
β	0.40	0.17	6.0 (94%)	3.6 (92%)	0.41	0.45
γ	0.48	0.13	12.2 (86%)	8.7 (89%)	0.39	0.37
δ	0.44	0.17	2.2 (100%)	2.4 (92%)	0.57	0.56
d1	0.80	0.44	8.84 (61%)	6.00 (65%)	0.192	0.209
d2	0.78	0.35	5.04 (87%)	4.01 (88%)	0.293	0.298
d3	0.81	0.38	9.06 (57%)	6.76 (59%)	0.134	0.141
d4	0.83	0.39	21.42 (15%)	15.55(22%)	0.019	0.028
Mar	0.044	0.007	456	451	0.000183	0.000210
Jun	0.046	0.006	380	361	0.000047	0.000057
Sep	0.046	0.006	414	415	0.000058	0.000074
Dec	0.049	0.006	403	395	0.000047	0.000059

Summary of Talk

- Temporal Graphs & Distance Metrics
  - Static shortest paths overestimate available hops and hence underestimate shortest path length
- Temporal Small World:
  - Contrary to intuition, slowly evolving graphs can be very efficient for data dissemination
- Future Work
  - Identifying important nodes
  - Malware propogation
    - Best nodes for patching
  - Spectral Analysis

Questions?

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project http://www.cl.cam.ac.uk/research/srg/netos/**spatialtemporalnetworks**

Further Reading

Small World Behavior in Time-Varying Graphs, J. Tang, S. Scellato, M. Musolesi, C. Mascolo, V. Latora, Physical Review E, Vol. 81 (5), 055101, May 2010.

Characterising Temporal Distance and Reachability in Mobile and Online Social Networks, J. Tang, M. Musolesi, C. Mascolo, V. Latora, ACM SIGCOMM Computer Communication Review (CCR). Vol. 40 (1), pp. 118-124. Jan 2010.

Temporal Distance Metrics for Social Network Analysis, J. Tang, M. Musolesi, C. Mascolo, V. Latora, In Proceedings of the 2nd ACM SIGCOMM Workshop on Online Social Networks (WOSN09). Aug 2009.