Mathematical Models of Invasive Species

Brian Lee  (Mathematician)

Prof. Rachel Norman & Paul Stebbing (CEFAS)
Department of Mathematics and Computing Science
University of Stirling
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Don’t get your hopes up
Invasive species: Why bother?

Yes or no?

- Second greatest threat to global biodiversity

- Contributing factor in the extinction of no less than 91 species

- Cost Great Britain approximately £1.7 billion
Examples (1)

The Good

Rhododendron

Coniferous Forests

Buddleias (Butterfly bush)

Rhizophagus grandis
Examples (2)

The Bad

Black rat (Rattus rattus)

Common/European rabbit

Kudzu
Examples (3)

The Downright Ugly

- **Signal Crayfish**
  - Native to North America
  - Carriers of *Aphanomyces astaci*
  - Mate with the native White-clawed crayfish
  - Culling thus far unsuccessful
The problem (1)

The Set-up

- Create a general model of trapping/removal
- Compare constant removal rate with both increasing and decreasing effort over time
- Investigate the effects of growth rate on “time to eradication”
The problem (2)

The Maths

- Start off with a basic logistic growth model: $\frac{dN}{dt} = rN(1 - \frac{N}{K})$
- Then add in a trapping rate: $\tau = -f(t)N$
- Evaluate the effects of $\tau$ on $N(t)$ with differing $f(t)$s

\[ f(t) = \begin{cases} kmt \ln(t+1) \delta(1 - 2\cos\left(\frac{2\pi(t-2.5)}{12}\right))\rho(10 + t/2 - 10\cos(2\pi t)) & \text{for } 0 \leq t \leq 300 \text{ and } k, m, \gamma, \delta, \rho \text{ constant} \end{cases} \]
The problem (3)

Trapping/removal functions

Constant

Decreasing - Log

Increasing – Straight Line

Seasonal

Increasing - Log

Increasing - Rastrigin
Results

- Constant
- Decreasing - Log
- Increasing – Straight Line
- Seasonal
- Increasing - Log
- Increasing - Rastrigin
Results II

But wait, there’s more

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Future work

Are we there yet?

- Biologically realistic model

- Look at effects on ecosystem at varying stages of invasion:
  - Signal crayfish – already here
  - King crab – starting to invade
  - Killer shrimp – not here yet

- Different stages, different maths
THANKS FOR LISTENING!