Survey of Interest Rate Models for Actuarial Use :

by

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Why:

- Actuarial products are over long contracts and interest rates are crucial in the valuation of the liabilities of a insurance provider.
- E.g a **life annuity** is a series of future payments in exchange of an immediate lump-sum payment.
- Or an **endowment policy** is a life insurance contract designed to pay a lump sum after a specified term (on its 'maturity') or on death.
- **At what interest rate is that payment going to be compounded at, and what interest rate does the insurer value that liability?????**

Why:

- What has been in use so far? **Fair Valuation.**
- Future interest rates are not known so the need for a model that can *forecast* future rates.
- Regulatory pressure for actuaries to use/consider the application of models from other areas of finance.

Definitions:

- A **bond** for example, is a series of payments from the issuer and a lump sum at the end of the term of the bond, a **zero coupon bond** will have no periodic payments but a single payment at maturity
- This has led to various models being developed for pricing and valuing financial products(interest rate derivatives)
- **Term structure models** these describe the behaviour of interest rates in the short-term(1month-10yrs) that is implied by zero coupon bonds.
- **Market models** developed from market observations of stocks

Properties of Ideal Models:

- As rates are unknown and random, will need a stochastic process
- Model to have economic interpretation, risk-neutral and arbitrage free
- Parsimonious whilst retaining the important features of the problem
- Transparent-can I explain how the model works in a few minutes?
- Development-can the model evolve and be implemented
- Examples of Term Structure models:
- **Vasicek (1977): $dr(t) = \alpha(\mu - r(t))dt + \sigma dW_t$**
- **Cox-Ingersoll-Ross (1985): $dr(t) = \alpha(\mu - r(t))dt + \sqrt{\sigma} dW_t$**
- **Hull and White (1990): $dr(t) = (\mu(t) - \alpha r(t))dt + \sigma dW_t$**

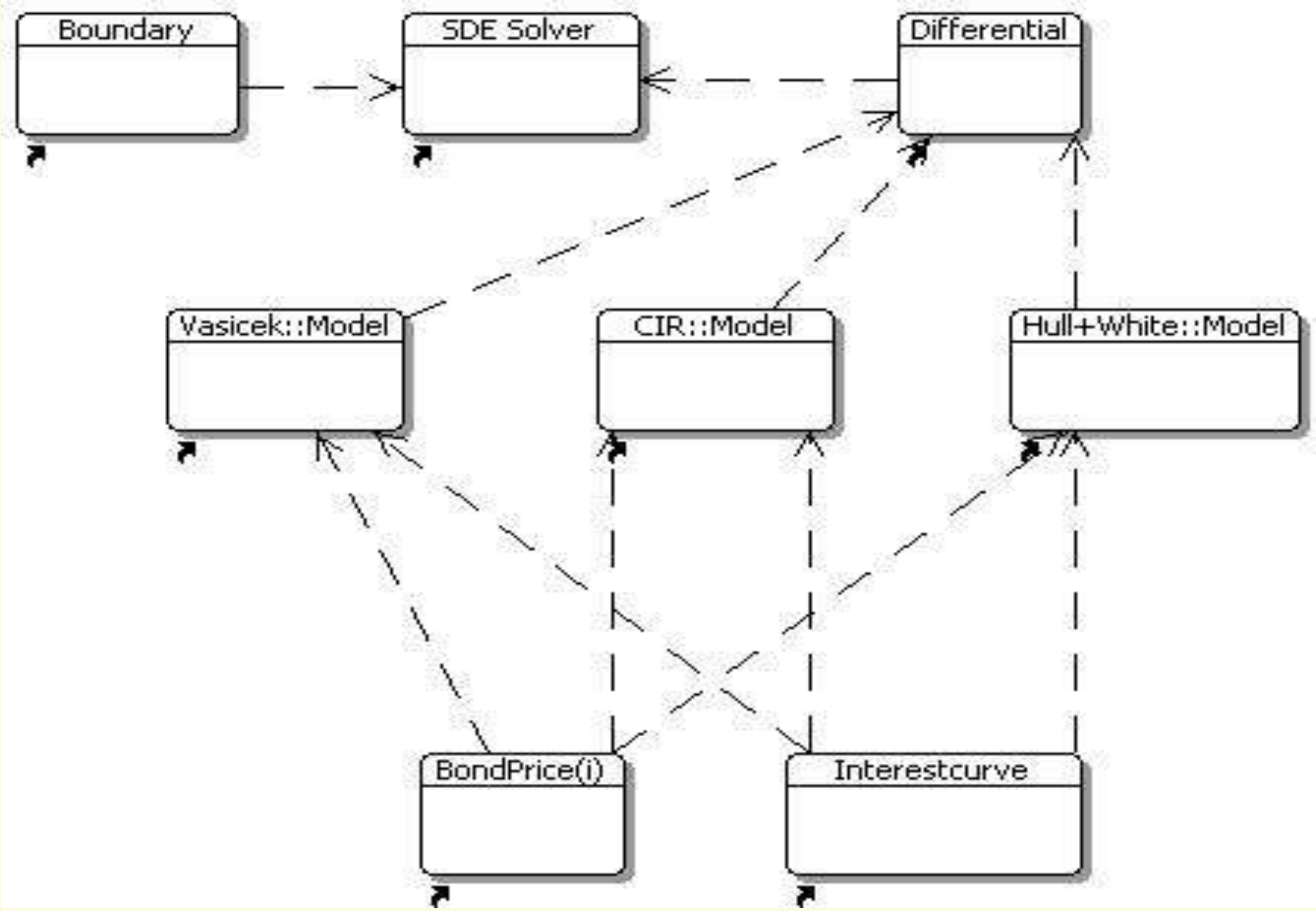
Approach to Solution

- The aim is to build a practical, simulation and risk-oriented framework for analysing interest rate models
- Will use Java as it will provide an intuitive interface and a graphical/visual description of the models
- Encapsulation of sometimes daunting mathematical formulae
- By use of classes and objects; models are of the form:

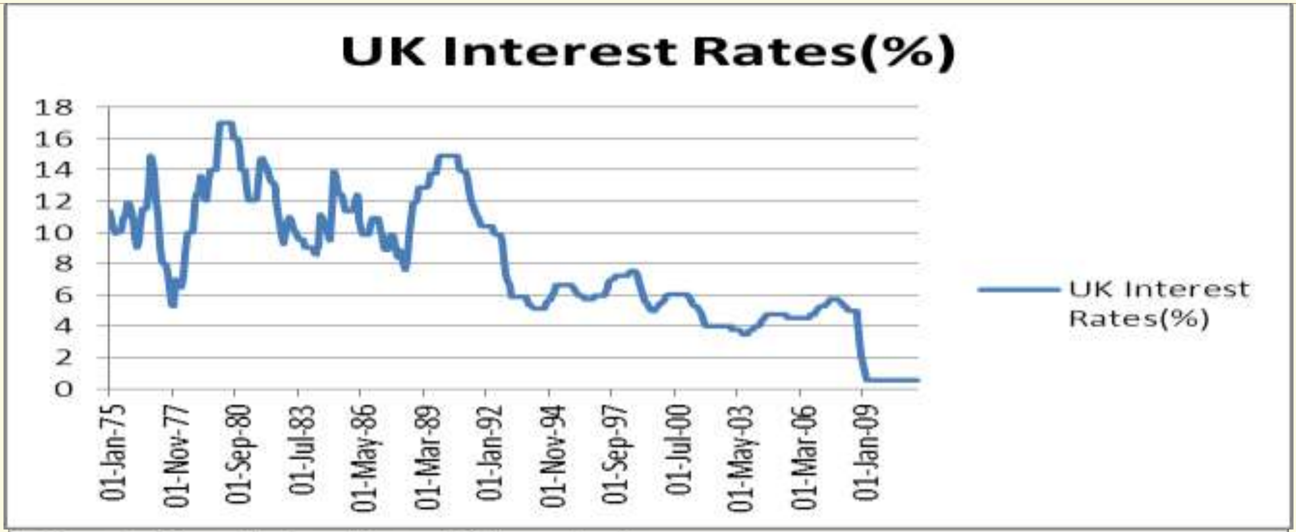
$$dr_t = a(r_t, t)dt + \sigma(r_t, t)dB_t$$

- Solve the equation only once

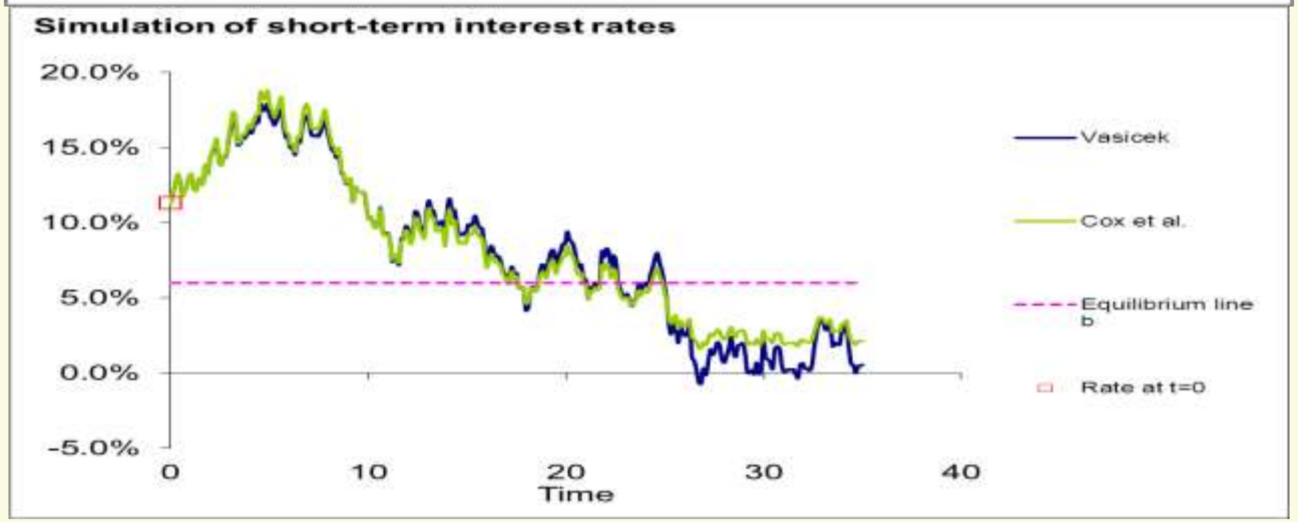
UML Class diagram:



Results:



UK Interest Rates(%)
18
16
14
12
10
8
6
4
2
0
01-Jan-75
01-Nov-77
01-Sep-80
01-Jul-83
01-May-86
01-Mar-89
01-Jan-92
01-Nov-94
01-Sep-97
01-Jul-00
01-May-03
01-Mar-06
01-Jan-09
UK Interest Rates(%)



Simulation of short-term interest rates
20.0%
15.0%
10.0%
5.0%
0.0%
-5.0%
0
10
20
30
40
Time
Vasicek
Cox et al.
Equilibrium line b
Rate at t=0

Uses:

- Gain an informed decision on which model to recommend for use
- Continued model development by adding more factors
- As times passes the parameters change and adjust model accordingly
- Use implied interest rates to value products and take on trading positions
- Any software solutions/applications – Economic scenario generator mostly in house.
- *In the hope that you have got, what I have failed to say…..*any Questions?