Fuzzy Sets and Fuzzy Logic

Another approach to reasoning about uncertainty, with a different mathematical basis, is *fuzzy logic.*

Brief history:

Standard classical (Boolean) logic (Aristotle, c 50BC; Boole, 1854) uses two possible *truth values:*

- A statement may be true (truth value 1) or false (truth value 0)

Łukasiewicz logic (early 20th century): three truth values:

- 2, 1 and 0 represent, respectively, "true", "false" and "unknown" or "irrelevant"
- This was further extended to an infinite-valued logic, where real numbers in the range [0,1] represent varying degrees of truth. Only of academic interest, until...

Lotfi Zadeh (1965) introduced *fuzzy set theory* and *fuzzy logic*, and promoted these as a way of reasoning about uncertainty in computer systems.

Fuzzy Sets

Here *properties* are represented by *fuzzy sets*

Properties might be like:

- temperature (fuzzy sets: *hot, warm, cool, cold*)
- height (fuzzy sets: *short, average, tall*)
- degree of danger (fuzzy sets: *safe, dangerous*)

For example, we might say

- the water is *cold*
- Jack is *tall*
- Rover is *dangerous*

In the context of a rule-based system, we might imagine asking ***'Is the water cold?' (Yes/No)***, and similarly for the other ones.

Or we might offer a menu of temperature choices (hot, warm, cool, cold)

Fuzzy Properties

Fuzzy set theory and fuzzy logic provide a precise, mathematical basis for reasoning about fuzzy sets and fuzzy properties.

In classical, 2-valued logic, we would have to distinguish **cold** from **not cold** by fixing a strict changeover point. We might decide that anything below 8 degrees Celsius is **cold**, and anything else is **not cold**. This can be rather arbitrary.

Fuzzy logic lets us avoid having to choose an arbitrary changeover point essentially by allowing a whole spectrum of '*degrees of coldness*'.

A set of temperatures like hot or cold is represented by a *function*. Given a temperature, the function will return a number representing the degree of membership of that set. This number is called a *fuzzy measure*.

Fuzzy Measures: Example

This example shows some fuzzy measures for the set cold:

Temp	Fuzzy measure	
-273	1	(cold)
-40	1	(cold)
0	0.9	(not quite cold)
5	0.7	(on the cold side)
10	0.3	(a bit cold)
15	0.1	(barely cold)
100	0	(not cold)
1000	0	(not cold)

Fuzzy Measures

The basis of the idea of 'coldness' may be how people use the word 'cold'. Perhaps

- 30% of people think that 10 degrees *C* is cold (function value 0.3), and
- 90% think that 0 degrees *C* is cold (function value 0.9).

Also it may depend on the context. In terms of the weather, 'cold' means one thing. In terms of the temperature of the coolant in a nuclear reactor 'cold' may mean something else, so we would need to have a 'cold' function appropriate to our context.

Fuzzy Sets

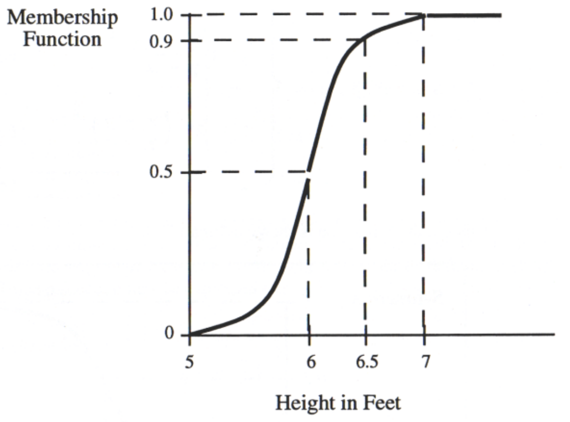
Fuzzy sets may be thought of as tables of values (as with 'cold' above), but often they are concerned with measurements that are made on a continuous scale.

So a fuzzy set is represented by a *graph* – the graph of the function which calculates, for each measurement value, the corresponding degree (in the range 0 to 1) to which the fuzzy property holds.

The opposite of "fuzzy" is "crisp". Crisp values are exact measurements (for example, the temperature recorded by a thermometer). The graph of a fuzzy set shows how the crisp values (on the x-axis) are assigned a fuzzy measure (on the y-axis) showing their degree of membership of the set.

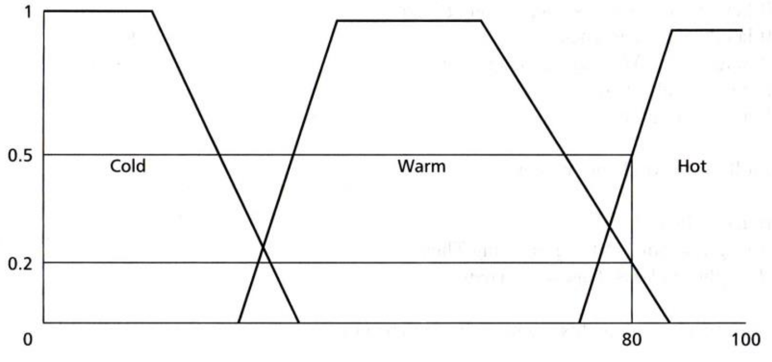
Fuzzy Sets

Tallness may be represented in such a way, perhaps via a graph like this:



Fuzzy Sets (2)

Here is a different graph showing three fuzzy sets. Notice the overlaps between cold and warm, and between warm and hot. 80 degrees is hot (0.5) and also warm (0.2).



Fuzzy Sets v. Probabilities

The values attached to properties in fuzzy logic are in some ways like probabilities, but it is clearly **not** probabilities that we are dealing with here.

We may know Jack's height exactly. The assertion 'Jack is tall (0.75)' measures how well Jack's height matches the sense of the word 'tall'.

On the other hand, 'the probability that Jack is tall is 0.75' would normally be used in a situation where we don't actually know Jack's height.

Fuzzy Sets in RBS

The benefit of all this is that the rules of an RBS can use (for example) the idea of 'coldness', rather than arbitrary changeover points involving specific values of temperature.

The most successful applications of fuzzy logic have been in automatic control systems (e.g., washing machines, air conditioners, high-end cameras…) The benefits include:

- Rules in fuzzy logic are clearer and simpler to understand.
- Fuzzy logic controllers can be more efficient and give smoother operation (e.g. Fuzzy air conditioner control vs traditional thermostat)

Our next example looks at a fuzzy logic controller for an automobile cruise control system…

Fuzzy Sets and Fuzzy Logic (Example)

Car cruise control. The driver selects a desired speed, and the control system is to maintain the speed by adjusting the throttle. See a video at

http://www.youtube.com/watch?v=wPlV2-OQqwg

The system can measure two things:

- **Speed error** (difference between actual speed and required speed).
- **Acceleration**

We shall have fuzzy sets associated with these:

- Speed Error Negative (SENeg)
- Speed Error Zero (SEZero)
- Speed Error Positive (SEPos)

Cruise Control

And

- Acceleration Negative (ANeg)
- Acceleration Zero (AZero)
- Acceleration Positive (APos)

And the system will exercise control over the **throttle**, with levels of throttle change represented by fuzzy sets:

- Throttle Reduce High Amount (RH)
- Throttle Reduce Small Amount (RS)
- Throttle Constant (C)
- Throttle Increase Small Amount (IS)
- Throttle Increase High Amount (IH)

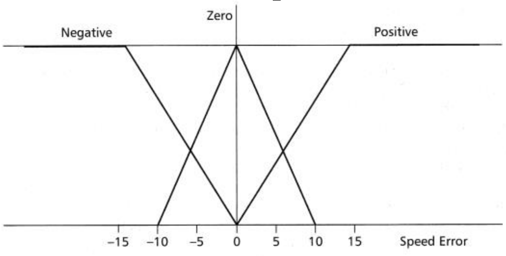
Cruise Control Fuzzy Sets

Speed Error:

Negative (SENeg)

Zero (SEZero)

Positive (SEPos)

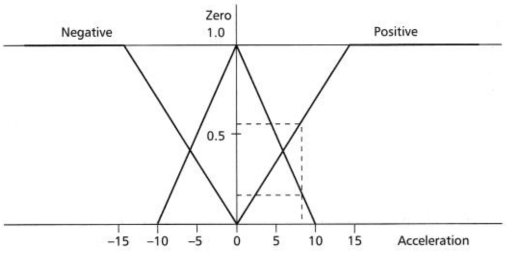


Acceleration:

Negative (ANeg)

Zero (AZero)

Positive (APos)



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Cruise Control Fuzzy Sets (2)

Throttle:

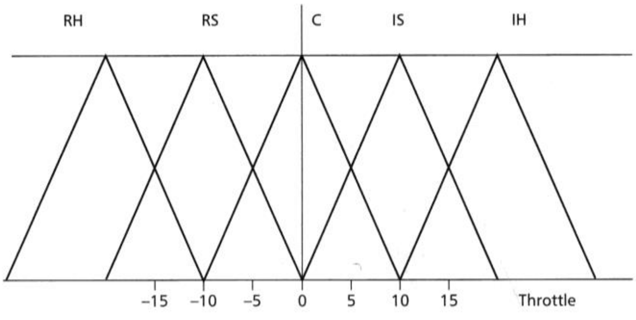
Reduce High Amount (RH)

Reduce Small Amount (RS)

Constant (C)

Increase Small Amount (IS)

Increase High Amount (IH)



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Cruise Control Rules

```
IF SEZero AND AZero THEN C
IF SEZero AND APos THEN RS
IF SEZero AND ANeg THEN IS
---------------------------
IF SENeg AND AZero THEN IS
IF SENeg AND APos THEN C
IF SENeg AND ANeg THEN IH
---------------------------
IF SEPos AND AZero THEN RS
IF SEPos AND APos THEN RH
IF SEPos AND ANeg THEN C
```


Fuzzy Logic

The cruise control system has exact (crisp) values for the speed error and the acceleration. Let's say the speed error is 0 and the acceleration is 8.

Inspection of our fuzzy set graphs then yields fuzzy values:

```
SEZero: 1.0     AZero: 0.2      APos: 0.6
```

(other fuzzy properties have value zero).

Which of our rules are applicable? Just those whose antecedents (IF parts) involve only these three properties:

```
IF SEZero AND AZero THEN C
IF SEZero AND APos THEN RS
```

Logical Combinations

In rules we may have AND combinations or OR combinations. In fuzzy logic we calculate values like this:

```
value(P AND Q) = min(value(P), value(Q))
value(P OR Q)  = max(value(P), value(Q))
```

For AND, take the smaller value,

For OR, take the larger value.

So in our example we calculate fuzzy values:

`SEZero AND AZero` has value `min(1,0.2)`, i.e. `0.2`

`SEZero AND APos` has value `min(1,0.6)`, i.e. `0.6`

Application of Rules

When we apply a rule, the fuzzy value for the antecedent is taken as the fuzzy value of the conclusion. So we get values:

`C: 0.2` `RS: 0.6`

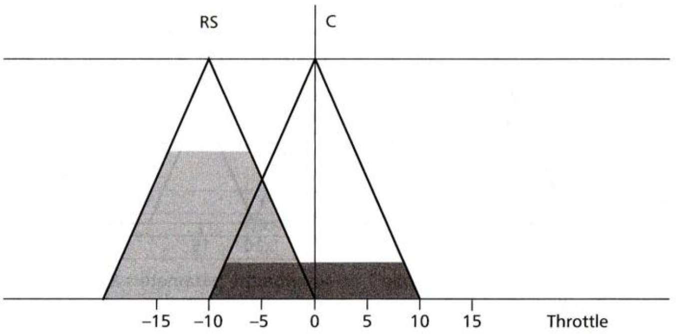
This suggests stronger support for RS (i.e. the action of reducing the throttle setting), but how can we decide exactly what action to take?

We need an exact ("crisp") value for the change to be made to the throttle setting.

To get this we use a process known as *defuzzification*...

Defuzzification

Consider the two fuzzy sets C and RS. In the diagram, the shaded areas are cut off at the levels 0.2 and 0.6. These represent the strength of each conclusion. The throttle setting required is found by calculating the 'centre of gravity' of the two shaded areas taken together.



Defuzzification (2)

The centre of gravity (or centroid) is the point on the x-axis at which a vertical line can be drawn that slices the shaded area into two equal areas.

The mathematics is standard (and not important for us).

In our example, the centroid is located at this x-value:

`(-10*8.4 + 0*3.6)/(8.4 + 3.6)`, i.e. `-7.0`

So the conclusion of our fuzzy reasoning process is that for the given values of the speed error and acceleration the throttle setting should be *reduced by 7.*

Hedges

There are some quite nice extensions to these ideas. In particular *hedges*. This means 'qualifying' words like *very*, *slightly*, or *somewhat*.

Let's imagine that we have a function *hot* which, when given a temperature, returns the degree of hotness. Then the function *very_hot:*

$$very\_hot(T) = (hot(T))^2$$

will have the meaning suggested by the name.

Similarly

$$not\_hot(T) = 1 - hot(T)$$

and

$$somewhat\_hot(T) = hot(T)^{0.5}$$

Rules using Hedges

Examples:

```
IF   Tree_distance is SOMEWHAT close
AND  Tree_angle is small_positive
THEN Turn slightly left
---------------------------------------------
IF   Tree_distance is SOMEWHAT close
AND  Tree_angle is small_negative
THEN Turn slightly right
---------------------------------------------
IF   Tree_distance is VERY close
AND  Tree_angle is zero
THEN Brake hard
```

Fuzzy Sets and Fuzzy Logic

Fuzzy logic can get rather mathematical.

When the mathematics takes over, intuitions may be lost.

It has been argued that fuzzy logic does not correspond with the way that people think.

But: it has been usefully applied, however, in situations where input is provided by sensors rather than people (e.g. Cameras, washing machines, climate control systems).

Further reading: Fuzzy Logic in wikipedia (reasonably simple)

http://videolectures.net/acai05_berthold_fl/ (starts simple!)

