# A NOVEL PERSPECTIVE INTO THE NEURONAL ENCODING ALONG THE RETINAL PATHWAY EMPLOYING TIMEFREQUENCY TRANSFORMATION: PART II — FOR COLOR 

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#### Abstract

This paper analyses how color information gets encoded along the retinal pathway in the time-frequency domain (TFD). A multinomial (multivariate polynomial) encoding for chromatic information is established based on the spatiotemporal spike trains corresponding to input stimuli varying in hue \& saturation. Simulation tools are used to generate the spike trains from the retinal layers. The Gabor time-frequency transformation presented in the companion paper part I [1] is employed here for analyzing the dominant harmonic variations corresponding to the color stimuli. For the set of color input stimuli considered, a bivariate polynomial encoding is observed with the order of the bivariate polynomial and its coefficients encoding the variations in amplitude of the dominant harmonics. Analysis is pursued along the similar lines presented in part I. The simulation results pertaining to encoding for color variations are presented. This paper suggests a new mathematical formulation called the generalized "stochastic multinomial" encoding, for the retinal pathway in general.


INDEX TERMS: Color Recognition, Dominant harmonics, Information processing, Neuronal encoding, Sensory pathways, Spatio-temporal analysis, Stochastic multinomial encoding, Visual pathway.

## 1 INTRODUCTION

One of the primary challenges in neuroscience is to decipher how information is encoded in the spike trains of neurons. The information processing in visual pathway is complex in comparison with other brain centers [2]. The pathway is involved in encoding many aspects of the visual stimulus like complex objects, complex scenes \& motions, and chromatic information.

This paper concentrates on an aspect of the information encoding along the visual pathway, the chromatic information, presenting simulation results up to the retinal pathway. The dominant harmonic analysis is performed using the algorithm 1 presented in part I. Chromatic information is analyzed on the basis of hue and saturation [3].

Section 2 enunciates the importance of color processing. Section 3 discusses the simulation results and the observed inferences. Section 4 gives the generalized multinomial encoding scheme for retinal pathway. Section 5 discusses the
need for structure prediction to analyze spatio-temporal information encoding in cortex and projects phase encoding \& stochastic multinomial encoding technique.

## 2 IMPORTANCE OF COLOR PROCESSING

There are infinite situations where the object recognition is based on color information. Color is one of the key factors in object \& natural scene perceptions, considered as the fourth dimension of vision [4]. Color processing is characterized by hue, saturation \& luminosity. In this paper, we focus on encoding of chromaticity of colored objects.

$$
\text { Chromaticity }=f \text { (hue, saturation) }
$$

Chromaticity is a function of hue and saturation [3]. Hue is an attribute associated with the dominant wavelength in a mixture of light waves. It represents the dominant color as perceived by the observer. Saturation refers to the relative purity or the amount of white light mixed with the hue. Luminosity gives a measure of the amount of energy an observer perceives from a light source. Luminosity can be modeled as a function of saturation alone.

## 3 SIMULATION RESULTS AND INFERENCE

Simulations are carried out for different hues and saturation. The colors considered are red, green, blue and grey. With respect to red, simulations are carried out by varying the saturation. Results are provided for 'within the layer' and 'across the different retinal layers'. The figures $1-4$ give the simulation results for different cases as listed in table 1.

Table 1 List of figures with description

| Figs. | Description |
| :--- | :--- |
| 1 a-h | Different colors across layers ${ }^{\uparrow}$ |
| 2 a-f | Different colors within layers |
| 3 a-h | Variations of Red across layers |
| 4 a-f | Variations of Red within layers |
| Horizontal, Bipolar \& Ganglion |  |

In figures $1-4$, the graph plots on the left show the variations in amplitudes of the dominant harmonics, and the plots on the right show the respective extrapolated bivariate polynomial (henceforth called polynomial). The corresponding input stimuli are inset in the graph plots on the

[^0]left. The tables $2-5$ show the extracted polynomial encoding functions $\&$ their coefficients corresponding to the graph plots presented in figures 1-4. These polynomials give the amplitude variations of the dominant harmonic components corresponding to the spatio-temporal neuronal activities for different color stimuli.

Encoding within and across layers is defined by the encoding format set $\mathrm{S}_{\mathrm{i}}$, containing the order ' O ' of the polynomial, and the coefficients of the polynomial function. The set $\mathrm{S}_{\mathrm{i}}$ contains the coefficients A, B, C, D, (refer tables $2-5)$ and $O$.

The core observation is that the encoding polynomial remains same for different colors (hues) 'across the layers' (table 2). The dominant harmonic variations are reflected through the polynomial coefficients; however the value of the element ' $O$ ' remains constant.

For different colors (hues), 'within the layers' (refer table 3 for values of $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{O}$ ), the encoding process is similar to 'across the layers', with the number of non-zero coefficients varying. The order of the polynomial varies for 'within' and 'across' the layers.

For different shades (saturation) of red color, the neuronal code 'within the layers' is given by the encoding format $\mathrm{S}_{\mathrm{i}}$. (Refer table 5 for element values). For encoding 'across the layers' table 4 gives the values of elements of the encoding format set $\mathrm{S}_{\mathrm{i}}$.

## 4 GENERALIZED MULTINOMIAL ENCODING FOR RETINAL PATHWAY

Object encoding is presented in part I, and color encoding in this paper, part II. The analysis for the sample input stimuli lead to the bivariate polynomial encoding, represented by the encoding format set Si , for both color and object.

For complex visual stimuli, it is very likely that the number of variables in the encoding polynomial function become greater than two, leading to multinomial (multivariate polynomial) encoding [5]. We propose a generalized multinomial encoding scheme ' $S$ ' incorporating the number of variables ' $N$ ' into the encoding format set Si proposed in part I \& II. The encoding format ' $S$ ' is given in terms of order of the multinomial, number of variables and the multinomial coefficients. The generalized multinomial encoding is as follows.

$$
\begin{aligned}
& \quad \mathrm{S}=\{\mathrm{O}, \mathrm{~N}, \mathrm{C}\} \\
& \text { Where } \\
& \mathrm{S} \text { - Generalized encoding (3 tuple) } \\
& \mathrm{O} \text { - Order of the encoding multinomial } \\
& \mathrm{N} \text { - Number of variables in the multinomial } \\
& \mathrm{C} \text { - Coefficient set. Each } \mathrm{C} \text { is a ' } \mathrm{k} \text { ' tuple. } \\
& \mathrm{k} \text { - Number of terms of the complete multinomial } \\
& \text { expression. }
\end{aligned}
$$

By complete multinomial, we mean the multinomial will have all terms inclusive of zero \& non-zero coefficients
generated as per the standard multinomial equation given below[5].

$$
\left(a_{1}+a_{2}+a_{3}+\ldots .,+a_{k}\right)^{n}=\sum_{\substack{n_{1}, n_{2} \ldots n_{k} \geq 0 \\ n_{1}+n_{2}+\ldots+n_{k}=n}} \frac{n!}{n_{1}!+n_{2}!+\ldots . n_{k}!} a_{1}^{n_{1}}+a_{2}^{n_{2}}+\ldots .+a_{k}^{n_{k}}
$$

The color encoding format \& object encoding format $\mathrm{S}_{\mathrm{i}}$ for the stimulus considered are a particular case of the generalized multinomial functional representation with the value of the element N as two.

## 5 DISCUSSION

The analysis of the complete encoding process involving the synapse (learning) is possible only when the interconnect structure of the neuronal population of the region of interest is available. Such interconnections for different brain centers can be predicted including dendrite, soma, pre and postsynaptic contacts [6]. With the availability of this interconnect structure the spatio-temporal neural encoding can be obtained by applying the same methodology presented in this paper.

With so much of research carrying out in fixing up the encoding process, still there is lot of scope left in understanding the information processing along the pathways. The authors feel that the neuronal information might not only get encoded in the amplitude of dominant harmonics but also in the phase of the dominant harmonics. The possibility of a phase encoding of neuronal information is being studied.

The generalized multinomial encoding presented in section 4 brings the element ' N ' into the encoding set. In analyzing the neuronal coding for complex visual scenes which have objects \& color frames striking the receptors in a random manner, the authors opine that each of the elements in the set ' $S$ ' will be under certain distribution and their $\mu$ and $\sigma$ values will depend on the respective elements.

This necessitates a mathematical formulation where each member of a set defining a multinomial is under some random distribution. The generalized multinomial encoding scheme can be extended to a generalized "stochastic multinomial" encoding. The authors suggest this new mathematical formulation, the generalized "stochastic multinomial" encoding, for neuronal encoding along retinal pathway when the input stimuli are complex visual scenes. Each element of set ' $S$ ' is now a random variable with the $\mu \& \sigma$ values of the elements encoding the information.

## 6 CONCLUSION

A bivariate polynomial for chromatic information encoding along the retinal pathway in the time-frequency domain (TFD) has been presented after analyzing spatiotemporal spike trains corresponding to input stimuli varying in hue \& saturation. A generalized multinomial encoding scheme incorporating the number of variables ' N ' is proposed for both color information encoding as well as object information encoding. A new mathematical formulation, the generalized
"stochastic multinomial" encoding has been suggested for analyzing neuronal information encoding for complex visual stimuli.

The dominant harmonic analysis to investigate the neuronal encoding process in the retinal pathway is totally a novel perspective put forth in papers I \& II. The dominant harmonic variations are modeled and brought under a new mathematical formulation; the stochastic multinomial function. This perspective can be applied to any neuronal layers in various brain regions.

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Figure 1 Simulation Results for different colors for 'across layers' (Contd..)


Figure 1 Simulation Results for different colors for 'across layers'


Figure 2 Responses of Horizontal, Bipolar and Ganglion for different colors for 'within layers' (contd...)


Figure 2 Responses of Horizontal, Bipolar and Ganglion for different colors for 'within layers'


Figure 3 Simulation Results for different saturation of red color for 'across layers' (contd..)


Figure 3 Simulation Results for different saturation of red color for 'across layers'


Figure 4 Responses of Horizontal, Bipolar and Ganglion for different saturation of red color for 'within layers' (contd...)


Figure 4 Responses of Horizontal, Bipolar and Ganglion for different saturation of red color for 'within layers'

Table 2 Polynomial and Coefficients from figure 1

| Stimulus | Polynomial | $A$ | $B$ | $C$ | $D$ |
| :--- | :--- | :---: | :---: | :---: | :---: |
|  | $\mathrm{~A}+\mathrm{B}^{*} \mathrm{X} 2+\mathrm{C}^{*} \mathrm{X} 1^{2}$ | $0.3253 \mathrm{E}+03$ | $0.2883 \mathrm{E}+00$ | $-0.9280 \mathrm{E}+01$ |  |
|  | $\mathrm{~A}+\mathrm{B}^{*} \mathrm{X} 2+\mathrm{C}^{*} \mathrm{X} 1^{2}$ | $0.7593 \mathrm{E}+03$ | $0.2511 \mathrm{E}-01$ | $-0.2125 \mathrm{E}+02$ |  |
|  | $\mathrm{~A}+\mathrm{B}^{*} \mathrm{X} 2+\mathrm{C}^{*} \mathrm{X} 1^{2}$ | $0.6105 \mathrm{E}+03$ | $-0.4327 \mathrm{E}-01$ | $-0.1599 \mathrm{E}+02$ |  |
|  | $\mathrm{~A}+\mathrm{B}^{*} \mathrm{X} 2+\mathrm{C}^{*} \mathrm{X} 1^{2}$ | $0.6367 \mathrm{E}+03$ | $-0.4175 \mathrm{E}+00$ | $-0.1864 \mathrm{E}+02$ |  |

Table 3 Polynomial and Coefficients from figure 2

| Layer | Polynomial | $A$ | $B$ | $C$ | $D$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Horizontal | $\mathrm{A}^{*} \mathrm{X1}^{2}+\mathrm{B} * \mathrm{X} 1+\mathrm{C} * \mathrm{X} 2+\mathrm{D}$ | $0.1959 \mathrm{E}+02$ | $-0.1663 \mathrm{E}+02$ | $0.9050 \mathrm{E}-01$ | $0.7126 \mathrm{E}+03$ |
| Bipolar | $\mathrm{A} * \mathrm{X}^{2}+\mathrm{B} * \mathrm{X} 1+\mathrm{C} 2+\mathrm{X} 2+\mathrm{D}$ | $-0.1543+\mathrm{E} 02$ | $-0.7060 \mathrm{E}+01$ | $0.1706 \mathrm{E}+00$ | $0.5736 \mathrm{E}+03$ |
| Ganglion | $\mathrm{A} * \mathrm{X1}^{2}+\mathrm{B} * \mathrm{X} 1+\mathrm{C} * \mathrm{X} 2+\mathrm{D}$ | $-0.1554 \mathrm{E}+02$ | $0.3664 \mathrm{E}+01$ | $0.4330 \mathrm{E}+00$ | $0.5028 \mathrm{E}+03$ |

Table 4 Polynomial and Coefficients from figure 3

| Stimulus | Polynomial | $A$ | $B$ | $C$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{~A}+\mathrm{B} * \mathrm{X} 2+\mathrm{C} * \mathrm{X} 1^{2}$ | $0.3253 \mathrm{E}+03$ | $0.2883 \mathrm{E}+00$ | $-0.9280 \mathrm{E}+01$ |  |
| $\mathrm{~A}+\mathrm{B} * \mathrm{X} 2+\mathrm{C} * \mathrm{X} 1^{2}$ | $0.5221 \mathrm{E}+03$ | $0.1648 \mathrm{E}-01$ | $-0.1388 \mathrm{E}+02$ |  |
| $\mathrm{~A}+\mathrm{B} * \mathrm{X} 2+\mathrm{C} * \mathrm{X} 1^{2}$ | $0.4968 \mathrm{E}+03$ | $0.1680 \mathrm{E}+00$ | $-0.1476 \mathrm{E}+02$ |  |
| $\mathrm{~A}+\mathrm{B} * \mathrm{X} 2+\mathrm{C} * \mathrm{X} 1^{2}$ | $0.5471 \mathrm{E}+03$ | $0.1915 \mathrm{E}+00$ | $-0.1347 \mathrm{E}+02$ |  |
| $\mathrm{~A}+\mathrm{B} * \mathrm{X} 2+\mathrm{C} * \mathrm{X} 1^{2}$ | $0.5113 \mathrm{E}+03$ | $0.2373 \mathrm{E}+00$ | $-0.1573 \mathrm{E}+02$ |  |

Table 5 Polynomial and Coefficients from figure 4

| Layer | Polynomial | $A$ | $B$ | $C$ | $D$ |
| :---: | :--- | :---: | :---: | :---: | :---: |
| Horizontal | $(\mathrm{A}+\mathrm{X} 2) /\left(\mathrm{B}+\mathrm{C}^{*} \mathrm{X1}^{2}\right)+\mathrm{D}$ | $-0.2425 \mathrm{E}+04$ | $0.9089 \mathrm{E}+01$ | $-0.1704 \mathrm{E}+00$ | $0.7490 \mathrm{E}+03$ |
| Bipolar | $(\mathrm{A}+\mathrm{X} 2) /\left(\mathrm{B}+\mathrm{C} * 1^{2}\right)+\mathrm{D}$ | $0.8517 \mathrm{E}+03$ | $0.2532 \mathrm{E}+02$ | $-0.1265 \mathrm{E}+01$ | $0.3117 \mathrm{E}+03$ |
| Ganglion | $\mathrm{A}^{*} \mathrm{X}^{2}+\mathrm{B}^{*} \mathrm{X} 1+\mathrm{C}^{*} \mathrm{X} 2+\mathrm{D}$ | $-0.1341 \mathrm{E}+02$ | $0.1665 \mathrm{E}+01$ | $0.8151 \mathrm{E}-01$ | $0.4307 \mathrm{E}+03$ |


[^0]:    ${ }^{1}$ www.warfindia.org , Authors listed in random order

