# A NOVEL PERSPECTIVE INTO THE NEURONAL ENCODING ALONG THE RETINAL PATHWAY EMPLOYING TIMEFREQUENCY TRANSFORMATION: PART I - FOR OBJECT 

N Venkateswaran<br>Director, Waran Research Foundation ${ }^{1}$, 46-B, Mahadevan Street, West Mambalam, Chennai - 600033, India warf@vsnl.net

R Chidambareswaran<br>Research Trainee, Waran Research Foundation, Chennai-600033, India<br>chids_raman@acm.org

B Harish<br>Research Trainee, Waran Research Foundation, Chennai-600033, India<br>harishbarath@hotmail.com


#### Abstract

This paper presents an innovative approach to investigate the information encoding taking place in the visual pathway, particularly the retinal pathway. Gabor time-frequency (TF) transformation is applied to the spatio-temporal spike trains from the layers of the retinal pathway, corresponding to input object stimuli varying in shape, orientation \& distance. These spike trains are generated by employing simulation tools. Using the TF transformation, a methodology is evolved to analyze information encoding in terms of dominant harmonic variations. Statistical analysis of these variations and extrapolation reveal that the dominant harmonic variations can be encoded as multinomial (multivariate polynomial) functions. For the set of input stimuli considered, a bivariate polynomial encoding is observed with the order of the polynomial and its coefficients encoding the variations in amplitude of dominant harmonics. Analysis of encoding is carried out for both 'within the layers', like horizontal only and 'across the layers', the entire retinal pathway. The simulation results presented enunciate the findings. In the companion paper part II, neuronal encoding for chromatic information is analyzed on similar lines. After analyzing the neuronal color encoding process in part II, a generalized encoding scheme applicable to object shapes as well as color is proposed.


INDEX TERMS: Dominant harmonics, Information processing, Neuronal encoding, Object Recognition, Sensory pathways, Spatio-temporal analysis, Visual pathway

## 1 INTRODUCTION

The retinal pathway comprises of layers of the photoreceptors, the horizontal, the bipolar and the ganglion cells. The Laterate Geniculate Nucleus (LGN) \& the Visual cortex further process the visual stimuli. These layers together with the striate \& extra striate regions in the brain work in unison for perception and recognition to take place. Information about stimulus undergoes processing at each stage starting from the receptor cells to the cortical regions. Each stage of the pathway encodes the information in a particular way leading to the recognition of objects in the cortex. Different schemes have been proposed [1] [2] [13] to understand the encoding process.

In papers part I \& II, a novel perspective into the neuronal encoding process of the retinal pathway is evolved. This is based on the dominant harmonic component analysis in TF domain, which has not yet been proposed. Unlike the
experimental analysis of the encoding process [3][12], this paper and the companion paper part II, present a simulation based approach and proposes an analytical model for object and color information encoding respectively.

Section 2 of this paper presents the retinal physiology. The proposed information encoding is discussed in section 3. The TFD analysis is dealt in section 4. Simulation results and inferences are provided in section 5 .

## 2 THE RETINAL PATHWAY

Various layers of the retina are briefly introduced in the following lines.

Rods- They are color insensitive but sensitive to dim light. Rods enable vision in dark-dim conditions. Cones- They are responsible for black, white and color vision and work in bright light. The Horizontal- The horizontal cells are responsible for sending visual information back to receptors through feedback synapses. These feedback responses are important to stabilize the frequency responses of the cones. The Bipolar- The pathway from the receptors to ganglion consists only of two synapses having two types of bipolar cells (ON and OFF center) in between. They have a center-surround organization similar to that of the ganglion. The Ganglion- These cells transmits information as trains of action potentials. This is unlike the photoreceptors, which respond to light with graded changes in membrane potential [4] [5].

The spikes from the ganglion cells of the retina are the final source of visual information passed to the brain. In this context, retinal processing is of great importance and is investigated here. Various physiological experiments with the retina have lead to interesting theories [6] that develop linear and nonlinear models for the ganglion cell activity.

## 3 PROPOSED INFORMATION ENCODING IN RETINAL PATHWAY

Conventional neuronal encoding schemes are classified into spike codes and rate codes [1] [2] [13]. Rate codes are based on mean firing rate as a method of encoding the information. The exact spike timing is lost in the process of taking the temporal average.

The spike train responses of the neuronal cells of the various layers are analyzed to understand how information is

[^0]encoded in the form of action potentials. The spatio-temporal pattern of pulses holds a key to the principle of neuronal information processing. This work aims to examine the dominant harmonic components of the spatio-temporal spike train in time-frequency domain (TFD) at each layer of the retinal pathway applying Gabor transformation. The effect of distance, orientation on the spatio-temporal encoding for object information is discussed. This approach examines how the input stimulus (the corresponding spatio-temporal spiking activities) gets encoded in the amplitude variations of the dominant harmonics in the Gabor TFD.

## 4 PROPOSED TFD ANALYSIS

### 4.1 GABOR TRANSFORMATION

The Gabor transformation [7] [8] decomposes a signal into functions localized in time and frequency. A common method for computing the transform coefficients involves the multiplication of a signal by a function, which is bi-orthogonal to a Gaussian window. The generalized Gabor expansion of a complex valued discrete signal is

$$
x(k)=\frac{1}{\sqrt{N}} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} a_{m, n} h(k-m N) e^{j \frac{2 \Pi n k}{N}}
$$

where $h(k)$ is a discrete periodic window sequence with period $\mathrm{L}=\mathrm{MN}$. The expansion coefficients are given by

$$
a_{m, n}=\frac{1}{\sqrt{N}} \sum_{k=0}^{L-1} x(k) \gamma(k-m N) e^{-j \frac{2 \Pi n k}{N}}
$$

where $\gamma(k) \quad$ satisfies the equation

$$
\sum_{k=0}^{L-1} h(k+m N) \gamma(k) e^{-j \frac{2 \Pi n k}{N}}=\delta_{m} \delta_{n}
$$

The Gaussian window shifted to the centre of the analysis window is used for Gabor transform.

$$
\begin{aligned}
& \quad h(k)=\left(\frac{\sqrt{2}}{N}\right)^{\frac{1}{2}} \exp \left[-\pi\left(\frac{k-(N-1) / 2}{N}\right)^{2}\right] \\
& \text { where }-\frac{M N}{2}+\frac{N-1}{2} \leq k \leq \frac{M N}{2}+\frac{N-1}{2}
\end{aligned}
$$

### 4.2 TF ANALYSIS

The TF analysis is a powerful tool employed extensively in digital and biological signal (EEG) processing and also in speech signal processing systems [9]. However it has not been applied to analyze the spatio-temporal spiking activities of the layers of the retinal pathway. This paper, part-I deals with the application of the Gabor transformation to analyze the spatiotemporal activities of different layers in the retinal pathway for object information encoding. This analysis is applied to evolve the encoding process within and along the different retinal layers in terms of the amplitude variations of dominant harmonics for varying input stimuli.

In fact there could be several dominant harmonics existing in a particular layer for a set of input stimuli. The set of

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dominant harmonics are analyzed, grouping them into different harmonic ranges.

### 4.3 STATISTICAL ANALYSIS

The dominant harmonics of the TF transformed signal are identified after statistical analysis using SAFNET (Statistical Analyzer For dominant harmonic analysis of Neuronal Encoding in TFD), which is developed as a part of this work. The different modules of SAFNET are shown in the Figure 1.


Figure 1: SAFNET (Statistical Analyzer For dominant harmonic analysis of Neuronal Encoding in TFD)
Mean $(\mu)$ is calculated by taking the range from the probabilistic value 0.5 of all the harmonics till the maximum probability of 1. The Standard Deviation ( $\sigma$ ) of the harmonics is calculated by the SDE. The DHE extracts the entire dominant harmonics having amplitudes higher than the $\mu+\sigma$. The DHT traces the amplitude variations of particular dominant harmonics across layers.

## 5 SIMULATION

The simulation model developed consists of layers modeling the photoreceptors, the horizontal, the bipolar \& the ganglion. Spatio-temporal spike train activities of the neuronal cells are obtained from each of these layers after presenting the stimulus over the photoreceptors. The spike trains are generated by employing simulation tools. The methodology of analyzing the spatio-temporal spike train of each layer is presented in Algorithm 1.

Algorithm 1: Analysis Methodology
Step1: Present the stimulus over the receptor Cells
Step2:Obtain the spatio-temporal spiking activity from each layer
Step3:Sample the spikes at discrete Intervals
Step4:Apply the time frequency Gabor transform to the sample values
Step5:Input the frequency components to SAFNET
Step6:Extract the amplitudes of the dominant harmonics from the Dominant harmonic tracer
Step7: Plot the extracted dominant harmonic Components
Step8: Perform multinomial curve fitting and identify the Order, Number of variables and the Coefficient set

### 5.1 RESULTS AND INFERENCES

Simulations are carried out for different object shapes varying the orientation and distance. The objects considered are cone, cube and donut. With respect to cone, simulations were carried out by varying orientation and with respect to cube by varying distance. Simulation results are provided for 'within the layer' and 'across the different retinal layers'.

The figures $2-7$, give the simulation results for different cases and are listed in table 1. In figures 2-7, the graph plots on the left show the variations in amplitudes of the dominant harmonics, and the plots on the right show the respective extrapolated bivariate polynomial (henceforth called polynomial). The corresponding input stimuli are inset in the graph plots on left.

Table 1 List of figures with description

| Figs. | Description |
| :--- | :--- |
| $2 \mathrm{a}-\mathrm{j}$ | Cone in different orientations ${ }^{\dagger}$ across <br> 3 layers <br> $4 \mathrm{a}-\mathrm{f}$ <br> $4 \mathrm{a}-\mathrm{j}$ |
| 5 Cone in different orientations within layers |  |
| $5 \mathrm{a}-\mathrm{f}$ | Cube at different distances ${ }^{\natural}$ across layers |
| $6 \mathrm{a}-\mathrm{b}$ | Donut across layers |
| $7 \mathrm{a}-\mathrm{f}$ | Shapes-cone, cube, donut within layers |

${ }^{\dagger}$ Angle varied by 10 deg (ori1, ori2....ori10) results shown for variations by 20 degs, ${ }^{\uparrow}$ Horizontal, Bipolar \& Ganglion, ${ }^{\text { }}$ Stimulus size varies with distance

The tables $2-6$ show the extracted polynomial encoding functions \& their coefficients corresponding to the graph plots presented in figures 2-7. These polynomial functions give the amplitude variations of the dominant harmonic components of the spatio-temporal neuronal activities for different input stimuli.

The core observation is that the order ' O ' of the encoding polynomial remains same across different object shapes both for 'within' (refer table 7) and 'across' the layers (refer table 6). The dominant harmonic variations are reflected through the polynomial coefficients. However, the order of the encoding polynomials remains constant for both 'within' and 'across' the layers for different object shapes.

We define an encoding format through set $S_{i}$ containing the order of the polynomial ' O ', and the coefficients of the polynomial function. The set $S_{i}$ contains the coefficients A, B, C , D ,(refer tables 2-7) and O . The encoding format is applicable for both 'within' and 'across' the layers.

For a cone with different orientations, the neuronal code 'within' the layers is given by the encoding format $S_{i}$ (Refer table 3 for values of the set elements). 'Across' the layers also the same encoding format is followed. For the coefficient values, and O of the polynomial, refer table 2.

The neuronal code 'within' the layers is given by $\mathrm{S}_{\mathrm{i}}$ (Refer table 5 for element values) for a cube placed at different distances from the receptors. Table 4 gives the element values for the set $\mathrm{S}_{\mathrm{i}}$ for the neuronal encoding 'across the layers' for a cube at varying distances from the receptors. The simulation
results for a donut placed at a particular skew, orientation \& distance are given in figure 6 .

Simulation Environment- Neuronal cells: 10,000+, Platform: P-IV, Tools: Matlab, Ret4, C, C++, LabFit, Sigmaplot, SAFNET.

## 6 DISCUSSION

The results presented in this paper are for individual objects. However the encoding process has to be analyzed in recognizing multiple objects like a pair of a cube $\&$ a pyramid placed together in different orientations. This needs to be extended to stationary and non-stationary complex visual sceneries.

An important issue to be addressed is the influence of synaptic learning process on the values of the elements of the encoding set $\mathrm{S}_{\mathrm{i}}$.

The authors feel that the encoding process related to complex object \& visual scenes recognition can be represented as a new generalized stochastic multinomial (multivariate polynomial) function discussed in the companion paper part II [10]. A stochastic multinomial function with the coefficients \& order of the polynomial as random variables might encode the random natural scenes. Each element of the encoding set might follow a probability distribution.

The multiple objects could be split into encoded basic components [11], which in turn will get represented by the stochastic multinomial function that may facilitate the cortex in the recognition process.

## 7 CONCLUSION

The Neuronal encoding process along the retinal pathway based on the dominant harmonic component analysis in TF domain has been presented in this paper for object information encoding. This TFD analysis can be extended to the visual cortex. Considering the complexity and quantum of information processing in the visual pathway, the methodology evolved here can be applied to other sensory pathways as well. Part II, presents further analysis on aspects of chromatic information encoding and projects a new mathematical formulation called the generalized "stochastic multinomial" encoding.

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Figure 2 Simulation results of different orientations of cone for 'across layers' (contd...)


Figure 2 Simulation results of different orientations of cone for 'across layers'


Figure 3 Response of horizontal, Bipolar \& Ganglion for different orientations of cone for 'within layers' (contd...)


Figure 3 Response of horizontal, Bipolar \& Ganglion for different orientations of cone for 'within layers'


Figure 4 Simulation Results for cube at different distances for 'across layers' (contd...)

|  | C | Coefficients $\mathrm{A}=.3006 \mathrm{E}+03 ; \mathrm{B}=0.3847 \mathrm{E}+01 ; \mathrm{C}=.8672 \mathrm{E}-01 ; \mathrm{D}=0.3552 \mathrm{E}+03$ |
| :---: | :---: | :---: |
|  | e | Coefficients $\mathrm{A}=0.6727 \mathrm{E}-02 ; \mathrm{B}=.5269 \mathrm{E}+01 ; \mathrm{C}=.9359 \mathrm{E}+01 ; \mathrm{D}=0.7514 \mathrm{E}+02$ |
|  |  | Coefficients $\mathrm{A}=0.3359 \mathrm{E}+03 ; \mathrm{B}=0.2119 \mathrm{E}+00 ; \mathrm{C}=.1101 \mathrm{E}+02$ |
|  | i | Coefficients $\mathrm{A}=0.3076 \mathrm{E}+03 ; \mathrm{B}=0.1272 \mathrm{E}+00 ; \mathrm{C}=-.7347 \mathrm{E}+01$ |

Figure 4 Simulation Results for cube at different distances for 'across layers'


Figure 5 Response of Horizontal, Bipolar \& Ganglion for a cube at different distances for 'within layers'


Figure 6 Simulation Results for donut for 'across layers'


Figure 7 Response of Horizontal, Bipolar \& Ganglion for different shapes (cone-Fig 2a, cube Fig 4g, donut Fig 6a) for 'within layers'

Table 2 Polynomial and Coefficients from Figure 2

| Stimulus | Polynomial | $A$ | $B$ | $C$ | $D$ |
| :--- | :--- | :---: | :---: | :---: | :---: |
| Cone ori 1 | $\mathrm{~A}+\mathrm{B}^{*} \mathrm{X} 2+\mathrm{C}^{*} \mathrm{X} 1^{2}$ | $0.6943 \mathrm{E}+03$ | $-0.1616 \mathrm{E}+00$ | $-0.2072 \mathrm{E}+02$ |  |
| Cone ori 2 | $\left(\mathrm{~A}+\mathrm{B}^{*} \mathrm{X} 2\right) /\left(1+\mathrm{C}-\mathrm{X} 1+\mathrm{D}^{*} \mathrm{X}^{2}\right)$ | $0.4024 \mathrm{E}+01$ | $0.2936 \mathrm{E}+00$ | $-0.6902 \mathrm{E}-03$ | $-0.3557 \mathrm{E}-01$ |
| Cone ori 3 | $(\mathrm{~A}+\mathrm{X} 1) /\left(\mathrm{B}+\mathrm{C}^{*} \mathrm{X}^{2}\right)+\mathrm{D}$ | $0.1618 \mathrm{E}+01$ | $-0.8605 \mathrm{E}+00$ | $0.2975 \mathrm{E}-05$ | $0.2086 \mathrm{E}+03$ |
| Cone ori 4 | $(\mathrm{~A}+\mathrm{X} 2) /\left(\mathrm{B}+\mathrm{C}^{*} \mathrm{X}^{2}\right)+\mathrm{D}$ | $0.3397 \mathrm{E}+06$ | $0.6540 \mathrm{E}+01$ | $0.1881 \mathrm{E}-02$ | $-0.5137 \mathrm{E}+05$ |
| Cone ori 5 | $\mathrm{~A} * \mathrm{X} 1+\mathrm{B} * \mathrm{X} 1^{2}+\mathrm{C}^{*} \mathrm{X} 2+\mathrm{D} * \mathrm{X} 2^{2}$ | $-0.5606 \mathrm{E}+01$ | $0.8329 \mathrm{E}+01$ | $0.5486 \mathrm{E}+01$ | $-0.7706 \mathrm{E}-02$ |

Table 3 Polynomial and Coefficients from Figure 3

| Layer | Polynomial | $A$ | $B$ | $C$ | $D$ |
| :--- | :--- | :---: | :---: | :---: | :---: |
| Horizontal | $\mathrm{A}+\mathrm{B}^{*} \mathrm{X} 2+\mathrm{C}^{*} \mathrm{X} 1^{2}$ | $0.7420 \mathrm{E}+03$ | $-0.3763 \mathrm{E}+00$ | $-0.2067 \mathrm{E}+02$ |  |
| Bipolar | $(\mathrm{A}+\mathrm{X} 2) /\left(\mathrm{B}+\mathrm{C} * \mathrm{X} 1^{2}\right)+\mathrm{D}$ | $-0.2262 \mathrm{E}+03$ | $0.4070 \mathrm{E}+01$ | $-0.9915 \mathrm{E}-01$ | $0.3998 \mathrm{E}+03$ |
| Ganglion | $\mathrm{A}^{2} \mathrm{X}^{2}+\mathrm{B} * \mathrm{X} 1+\mathrm{C}^{*} \mathrm{X} 2+\mathrm{D}$ | $0.5254 \mathrm{E}+01$ | $-0.1342 \mathrm{E}+02$ | $0.1458 \mathrm{E}+00$ | $-0.1003 \mathrm{E}+03$ |

Table 4 Polynomial and Coefficients from Figure 4

| Stimulus | Polynomial | $A$ | $B$ | $C$ | $D$ |
| :---: | :--- | :---: | :---: | :---: | :---: |
| Distance 1 | $\mathrm{A}^{*} \mathrm{X} 1+\mathrm{B} * \mathrm{X} 2+\mathrm{C} / \mathrm{X} 1^{4}$ | $-0.6223 \mathrm{E}-05$ | $-0.7231 \mathrm{E}-06$ | $0.2773 \mathrm{E}-03$ |  |
| Distance 2 | $(\mathrm{A}+\mathrm{X} 2) /\left(\mathrm{B}+\mathrm{C} * \mathrm{X} 1^{2}\right)+\mathrm{D}$ | $-0.3006 \mathrm{E}+03$ | $0.3847 \mathrm{E}+01$ | $-0.8672 \mathrm{E}-01$ | $0.3552 \mathrm{E}+03$ |
| Distance 3 | $\mathrm{A}^{*} \mathrm{X} 2^{2}+\mathrm{B}^{*} \mathrm{X} 2+\mathrm{C}^{*} \mathrm{X} 1+\mathrm{D}$ | $0.6727 \mathrm{E}-02$ | $-0.5269 \mathrm{E}+01$ | $-0.9359 \mathrm{E}+01$ | $0.7514 \mathrm{E}+02$ |
| Distance 4 | $\mathrm{A}+\mathrm{B} * \mathrm{X} 2+\mathrm{C} * 1^{2}$ | $0.3359 \mathrm{E}+03$ | $0.2119 \mathrm{E}+00$ | $-0.1101 \mathrm{E}+02$ |  |
| Distance 5 | $\mathrm{A}+\mathrm{B}^{*} \mathrm{X} 2+\mathrm{C} * \mathrm{X} 1^{2}$ | $0.3076 \mathrm{E}+03$ | $0.1272 \mathrm{E}-+00$ | $-0.7347 \mathrm{E}+01$ |  |

Table 5 Polynomial and Coefficients from Figure 5

| Layer | Polynomial | $A$ | $B$ | $C$ | $D$ |
| :--- | :--- | :---: | :---: | :---: | :---: |
| Horizontal | $(\mathrm{A}+\mathrm{X} 1) /\left(\mathrm{B}+\mathrm{C}^{*} \mathrm{X} 2^{2}\right)+\mathrm{D}-\mathrm{X} 2$ | $-0.1355 \mathrm{E}+07$ | $0.9434 \mathrm{E}+06$ | $-0.2021 \mathrm{E}+01$ | $0.7772 \mathrm{E}+00$ |
| Bipolar | $\mathrm{A}+\mathrm{B}^{*}{\mathrm{X} 2+\mathrm{C}^{*} \mathrm{X} 1^{2}}^{2}$ | $0.3336 \mathrm{E}+03$ | $0.2129 \mathrm{E}+00$ | $-0.9332 \mathrm{E}+01$ |  |
| Ganglion | $\mathrm{A}^{*} \mathrm{X}^{2}+\mathrm{B} * \mathrm{X} 2+\mathrm{C} * \mathrm{X} 1+\mathrm{D}$ | $-0.9682 \mathrm{E}-02$ | $0.7094 \mathrm{E}+01$ | $-0.2299 \mathrm{E}+01$ | $0.1273 \mathrm{E}+03$ |

Table 6 Polynomial and Coefficients from Fig 2a, Fig 4g and Fig 6

| Shape | Polynomial | $A$ | $B$ | $C$ | $D$ |
| :--- | :--- | :---: | :---: | :---: | :---: |
| Cone | $\mathrm{A}+\mathrm{B}^{*} \mathrm{X} 2+\mathrm{C} * \mathrm{X} 1^{2}$ | $0.6943 \mathrm{E}+03$ | $-0.1616 \mathrm{E}+00$ | $-0.2072 \mathrm{E}+02$ |  |
| Cube | $\mathrm{A}+\mathrm{B} * \mathrm{X} 2+\mathrm{C}^{*} \mathrm{X} 1^{2}$ | $0.3359 \mathrm{E}+03$ | $0.2119 \mathrm{E}+00$ | $-0.1101 \mathrm{E}+02$ |  |
| Donut | $\mathrm{A}+\mathrm{B}^{*} \mathrm{X} 2+\mathrm{C} * \mathrm{X} 1^{2}$ | $0.4119 \mathrm{E}+03$ | $0.7110 \mathrm{E}-01$ | $-0.1166 \mathrm{E}+02$ |  |

Table 7 Polynomial and Coefficients from Figure 7

| Layer | Polynomial | $A$ | $B$ | $C$ | $D$ |
| :--- | :--- | :---: | :---: | :---: | :---: |
| Horizontal | $\mathrm{A}+\mathrm{B}^{*} \mathrm{X} 2+\mathrm{C}^{*} \mathrm{X1}^{2}$ | $0.6337 \mathrm{E}+03$ | $-0.6156 \mathrm{E}+00$ | $-0.1411 \mathrm{E}+02$ |  |
| Bipolar | $\mathrm{A}+\mathrm{B}^{*} \mathrm{X} 2+\mathrm{C}^{*} 1^{2}$ | $0.6139 \mathrm{E}+03$ | $-0.3217 \mathrm{E}+00$ | $-0.1758 \mathrm{E}+02$ |  |
| Ganglion | $\mathrm{A}^{*} \mathrm{X} 2^{2}+\mathrm{B}^{*} \mathrm{X} 2+\mathrm{C}^{*} * \mathrm{X} 1+\mathrm{D}$ | $-0.9232 \mathrm{E}-02$ | $-0.7495 \mathrm{E}+01$ | $0.5836 \mathrm{E}+01$ | $0.2868 \mathrm{E}+03$ |


[^0]:    ${ }^{1}$ www.warfindia.org, Authors listed at random order

