

Controlling the cortex state transitions by altering the oscillation energy

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Abstract

The method of controlling dynamical systems based on alteration of the oscillation energy is applied to multistable systems, attractors of which are qualitatively similar to their counterparts in the cortex. The attraction basins of “basal” and “epileptic” states are associated with their own energy levels. Changing the system energy causes switching from the pathological attractor to the physiological one.

The approach utilizes simple feedback depending solely on the output signal and, hence, is especially useful when the system parameters are inaccessible or are costly to adjust, which is characteristic of biological systems. The technique requires neither knowledge of the system equations nor computation of the control signal, and, hence, can be useful for control as well as identification of unknown systems.

1 Introduction

The cortex is an active system producing a range of the oscillation patterns. In a rest, the cortex produces so-called spontaneous background activity. This is a low energy state characterized by irregular low amplitude oscillations demonstrating signs of chaotic behaviour. Indeed, the analysis of electroencephalograms (EEGs) reveals their underlying spatiotemporal structure and predictability to some extent, which is manifestation of deterministic chaos [1]. Unlike, the noise or stochastic dynamics is unstructured and unpredictable. Chaotic activity is revealed in both single cell and electroencephalographic recordings.

Normal physiological dynamics of the cortex can undergo abrupt transition to an alternative state characterized by high amplitude oscillations that are much more regular and correlated than the back-

ground ones. This is so-called seizure state corresponding to some form of epilepsy.

From the dynamical system theory view, there are two possible main scenarios of the above state transition. First one assumes that the basal state is the only attractor of the physiological cortical dynamics. Pathological changing the system parameters leads to the birth of another alternative attracting state, the epileptic one, whereas the basal state becomes the repelling one. Second scenario assumes that such a complex system like cortex consisting of billions of neurons can possess multiple attractors with own basins of attraction at the given parameters. The basal state is supposed to have the largest basin to guarantee the system robustness. The epileptic and basal attractors are the coexisting ones. This case, of course, more complex than the previous one, and, itself can be divided on three possible subscenarios. In first subscenario, the cortex parameters can remain unchanged since an excessive pulse (say, a pulse coming from another parts of the brain) may switch the trajectory to another basin of attraction. Second subscenario assumes that changing the cortex parameters results in enlargement of the pathological basins. In this case, normal physiological pulses can switch system to the pathological state. And finally, the third subscenario assumes that initially only physiological basal attractor exists, and pathological changing the cortex parameters can lead to appearance of coexisting pathological attractors. In terms of Poisson stability, chaotic attractor is a stable set, and two hypothesis can be distinguished as the monostable and multistable ones. Throughout the paper, we refer to these hypotheses as above.

In this paper, we consider controlling the generalized Chua’s circuit operating in the parameter region with coexisting chaotic and periodic attractors. We consider single Chua’s cell as well as diffusively coupled ring of the these cells. For these systems, the signals measured are claimed to be qualitatively

similar to some EEG measurements in biology [2].

At first sight, the simplest approach to control multistable system is to apply a large enough pulse to move the trajectory out of the undesired region. However, it is naive to think so. Indeed, complex system can have many basins of attractions which, in turn, can have very complicated shapes. The strength of the pulse should, thus, be carefully calculated in advance.

Recently, the technique for controlling multistability was developed that is based on applying slowly varying perturbation added to the system parameter [4]. For coexisting periodic attractors of the Hénon map and the laser rate equations, the elimination of the higher period attractors was shown. Under perturbation, the higher period state becomes chaotic via the period doubling route and the chaotic state undergoes boundary crisis. The system, thus, becomes monostable. The results are verified in experiments with the cavity loss modulated CO₂ laser. In relation to dynamical diseases, such as epilepsy or Parkinsonian tremor, one need, in opposite, to eliminate near-period-1 state and preserve the state with infinite period, the chaotic one. Further research is necessary to investigate the applicability of the technique to different types of coexisting attractors and to different systems.

In this paper, we applied another recently developed method for controlling dynamical systems. It is based on alteration of the system oscillation energy [5]. The main idea is that every attractor is associated to own energy. When the system energy matches the definite level, the trajectory follows the attractor associated to this level of the energy. By changing the energy, one can, thus, control the system behaviour. For example, one can stabilize the repellors by altering the system energy. For multistable systems, basin of attraction is associated with its own energy. As shown in this paper, changing the system energy causes switching from one basin of attraction to another. For the cortex dynamics, the transition from basal state to epileptic one is always the transition from lower to higher energy characterized by significant increase of the latter. Our aim is, thus, to decrease the cortex energy to restore the cortex normal mode.

In this paper, we first consider the controlling general oscillator. Then, we apply the technique to multistable systems, attractors of which are qualitatively similar to their counterparts in the cortex. In these systems, we control the transition from "undesired" attractor to "desired" one. Finally, we discuss controlling globally and locally coupled networks and possible competition of the attractors in the latter.

2 General approach

Let us consider controlling a general type nonlinear oscillator

$$\ddot{x} + \chi(x, \dot{x}) + \xi(x) = F(t) + g(x, \dot{x}) \quad (1)$$

where $\chi(x, \dot{x})$, $\xi(x)$ and $g(x, \dot{x})$ are dissipative or energy-generating component, restoring force, and control force, respectively. These functions are nonlinear in general case. Also, $\chi(x, \dot{x})$ and $g(x, \dot{x})$ are assumed not contain an additive function of x . $F(t)$ is an external time-dependent driving force.

At $F(t) = 0$ and $g(x, \dot{x}) = 0$, Eq. (1) possesses the equilibriums defined by the equation $\xi(x) = 0$. In the oscillators with nonlinear damping (say, van der Pol and Reyleigh oscillators), the equilibrium becomes unstable at some parameter values, and stable self-sustained oscillations are excited. In other types of oscillators, say Duffing oscillator, the limit cycle arises under the action of the periodic driving force. We assume that at some driving amplitudes, the limit cycle becomes saddle, and a new attractor, say a period-2 cycle, arises. In many well-known example, this scenario leads, through sequence of bifurcations, to the birth of chaotic attractor.

One can define an energy of oscillations as a sum of the "potential" energy and "kinetic" energy

$$E(t) = \int \xi(x) dx + \frac{1}{2} \dot{x}^2 \quad (2)$$

and an averaged over the period T energy

$$\langle E \rangle = \frac{1}{T} \int_0^T \left(\int \xi(x) dx + \frac{1}{2} \dot{x}^2 \right) dt \quad (3)$$

For periodic dynamics T is the oscillation period, and for chaotic one $T \rightarrow \infty$. Each behavior of the oscillator is assigned to the value of the averaged energy (3). If the oscillation amplitude is sufficiently small, the limit cycle oscillations can be approximated as $x \simeq \rho \sin \omega t$, which gives $\langle E \rangle = \frac{1}{2} \rho^2$.

The following control strategy can be proposed. Starting at the lower energy attractor, one stabilizes the higher energy repellors by sequential increasing the averaged oscillation energy. On the contrary, decreasing this energy leads to stabilization of the lower energy repellors.

The change of the energy (2) yields

$$\begin{aligned} \dot{E}(t) &= \xi(x) \dot{x} + \dot{x} \ddot{x} \\ &= (-\chi(x, \dot{x}) + F(t) + g(x, \dot{x})) \dot{x} \end{aligned} \quad (4)$$

The last term of (4) represents the energy change caused solely by the control. We require that

$$g(x, \dot{x}) \dot{x} > 0 \quad (< 0) \quad (5)$$

for $\forall(x, \dot{x})$. The minimal feedback satisfying (5) is achieved at $g = g(\dot{x})$. Indeed, a simple linear (relative to the velocity) control $g(\dot{x}) \sim \dot{x}$ suffices. However, this type of control as well as nonlinear controls of higher power, say $g(\dot{x}) \sim \dot{x}^3$ can lead to undesirable instabilities in the system. Therefore, the controller dynamics should be described by the bounded functions. In this paper, we consider

$$g(\dot{x}) = k h(\dot{x}) \quad (6)$$

where

$$h(\dot{x}) \begin{cases} > 0 (\rightarrow a), & \text{if } \dot{x} > 0 (\rightarrow \infty) \\ = 0, & \text{if } \dot{x} = 0 \\ < 0 (\rightarrow -a), & \text{if } \dot{x} < 0 (\rightarrow -\infty) \end{cases} \quad (7)$$

with $a > 0$ and finite. The $h(\dot{x}) = -h(-\dot{x})$, i.e. it is assumed to be odd. Throughout, we consider $g(\dot{x}) = k \tanh(\beta \dot{x})$ with $0 < \beta \leq \infty$ determining the function slope.

The perturbation (6-7) is specially tuned to control the equilibriums — their positions are not changed by the control as it vanishes at $\dot{x} = 0$. $\dot{E} = 0$ at the equilibriums respectively. The above control does not vanish at the dynamic attractors. Our aim, however, is not stabilization of the UPOs of the unperturbed system existing at given parameter values, but rather shift the system into the region of the desired behavior (say, stable or unstable region if one requires to stabilize or destabilizes the system, respectively). The energy (3) will be changed so as to much the energy of the desired stable orbit or chaotic attractor, respectively.

The control (6-7) does not depend on type of the function $\chi(x, \dot{x})$, $\xi(x)$, and $F(t)$, and, hence, can be applied to linear and nonlinear oscillators, to regular and chaotic dynamics.

The approach can be generalized to the case of coupled oscillator networks. Generally, the oscillators and their connections are assumed to be non-identical. In this case, the motion equations yield

$$\begin{aligned} \ddot{x}_i + \chi_i(x_i, \dot{x}_i) + \xi_i(x_i) + Q_i[x_i] \\ = F_i(t) + g_i(x_i, \dot{x}_i) \end{aligned} \quad (8)$$

where $i = 1, \dots, n$ and Q_i is the coupling operator acting on i -th oscillator. The coupling can be local (say, via diffusion when $Q_i = \alpha_i(x_{i-1} - 2x_i + x_{i+1})$, where α_i is the coupling strength of i -th oscillator), global (say, via mean field when $Q_i = \alpha_i Q = \alpha_i \frac{1}{n} \sum_{i=1}^n x_i$), or intermediate-range one (say, via modified mean field that includes the interaction range). Also, some combinations of the above exist (say, via randomly adding a fraction of connections in the originally nearest-neighbor coupled network forming, thus, small-world coupled network). The

expression for energy of i -th oscillator

$$E_i(t) = \int (\xi_i(x_i) + Q_i[x_i]) dx_i + \frac{1}{2} \dot{x}_i^2 \quad (9)$$

results in the energy change

$$\begin{aligned} \dot{E}_i &= (\xi_i(x_i) + Q_i[x_i]) \dot{x}_i + \dot{x}_i \ddot{x}_i \\ &= (-\chi_i(x_i, \dot{x}_i) + F_i(t) + g_i(x_i, \dot{x}_i)) \dot{x}_i \end{aligned} \quad (10)$$

that is equivalent to (4). Thus, the controlling strategy developed for a single oscillator is applicable to the oscillator networks.

Application of the approach to 3-D autonomous dynamical systems presented in oscillatory form is considered below.

3 Controlling Chua's cells

3.1 Single cell dynamics

Let the qualitative dynamics of the cortex transitions to be modelled by the generalized Chua's circuit equations [2]:

$$\begin{aligned} \dot{x} &= a(y - f(x)) \\ \dot{y} &= x - y + z \\ \dot{z} &= -by \end{aligned} \quad (11)$$

where

$$f(x) = m_3 x + \frac{1}{2} \sum_{j=1}^3 ((m_{j-1} - m_j)(|x + c_j| - |x - c_j|)),$$

a, b, m_p, c_s , ($p = 0, 1, 2, 3$; $s = 1, 2, 3$) are the system parameters.

Substituting $y = -\dot{z}/b$ to the second equation of (11), obtain

$$\begin{aligned} \ddot{z} + \dot{z} + bz &= -bx \\ \dot{x} + af(x) &= -\frac{a}{b} \dot{z} \end{aligned} \quad (12)$$

To apply the above approach, one need to add the feedback $g(\dot{z})$ to the first equation of system (12). For the above oscillator, the change of the energy (2) caused by this control yields $\dot{z}g(\dot{z})$. If $g(\dot{z})$ takes the form (6-7), the latter term always provides the increase (decrease) of the oscillation energy for positive (negative) perturbation magnitudes. We, thus, consider $g(\dot{z}) = k \tanh(\beta \dot{z})$. Taking the inverse change of the variables $\dot{z} = -by$, obtain the control feedback to be applied to second equation of the system (11):

$$g(y) = -k \tanh(\tilde{\beta} y), \quad (13)$$

where $\tilde{\beta} = \beta b$.

Let take $a = 9$, $b = 14.3$, $m_0 = -0.14$, $m_1 = 0.28$, $m_2 = -0.21$, $m_3 = 0.1$, $c_1 = 1$, $c_2 = 3$, and $c_3 = 5$. For these parameters, the unperturbed cell possesses coexisting chaotic and periodic attractors. The periodic regime is characterized by high-amplitude oscillations, while the chaotic oscillations are low-amplitude ones. We associate the former and the later with abnormal or pathological behaviour (such as epileptic seizure or Parkinsonian tremor) and normal one (i.e. system's basal state), respectively.

Let the initial conditions to be chosen so as the trajectory attracted to the periodic orbit. The system dynamics is, thus, pathological one. Our aim is to switch the dynamics back to its normal basal state. Application of the perturbation (13) with positive k to the system results in decreasing its oscillation energy. The system is forced, thus, to regimes of lower oscillation energy, i.e. low amplitude chaos in our case. Indeed, at sufficient magnitude of k , switching from high amplitude periodic attractor to low amplitude chaotic one occurs (Fig. 1(a), $50 \leq t \leq 250$).

Every attractor (the periodic and chaotic one) possesses own basin of attraction in system's state space. Turning the perturbation off does not switch the attractors - the system remains in the basin of chaotic attractor - but increases the dimension of chaotic attractor (see Fig. 1(a) at $t \geq 250$). That is, the oscillations becomes even less regular with the amplitudes changing in a broader range. The latter attractor is the invariant set of the system (11), i.e. it is its natural (physiological) attractor, whereas the former one is a feature of the perturbed system. Projection of these attractors to space (x , y) clearly shows the difference - the chaotic attractor induced by control belongs to the Rössler attractor class whereas the attractor of unperturbed system is Chua's double-scroll attractor.

The perturbation (13) is specially tuned to control of the system (11) fixed points. Indeed, at every fixed point of the system (11) (i.e. at $(1.5, 0, -1.5)$, $(0, 0, 0)$, or $(-1.5, 0, 1.5)$), the perturbation is vanishing. The latter means that stabilized fixed points are invariants of the unperturbed system, i.e. its intrinsic features. Further increasing k leads to decreasing the oscillation amplitude and, eventually, to suppression of the oscillations and stabilization of the fixed point $(1.5; 0; -1.5)$. This fixed point corresponds to the controlled locus of the double scroll (the Rössler type attractor in Fig. 1). Let us fix k but increase β instead. At sufficient slope value, the above fixed point is stabilized (Fig. 1(b)).

For a brain dynamics, however, fixed points correspond to undesirable behaviour, since they are associated with the deep anesthesia state.

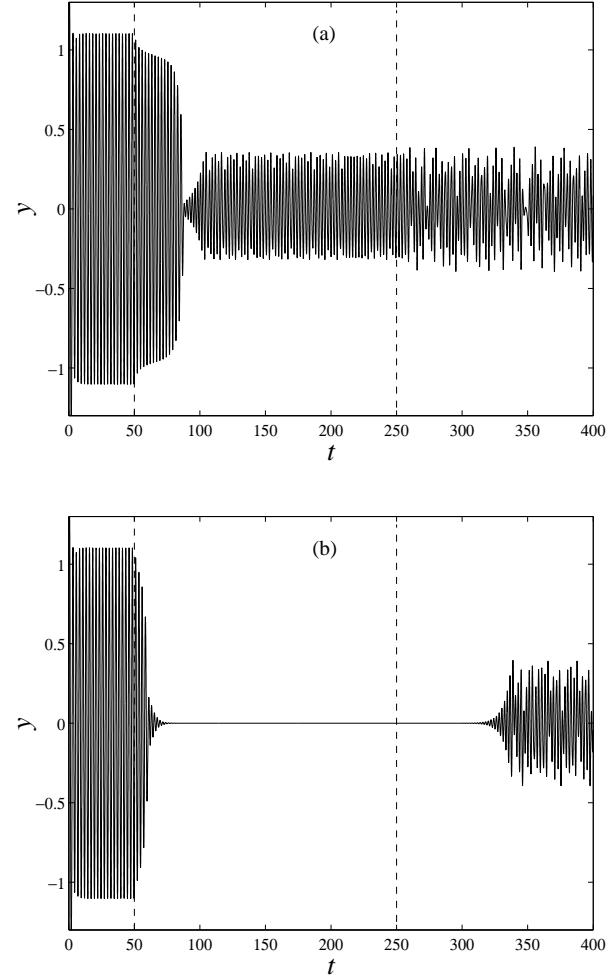


Figure 1: Dynamics of the state variable y of the system (11)-(13) at (a): $\tilde{\beta} = 1$, $k = 0$ ($250 \leq t < 50$); $k = 0.135$ ($t \geq 100$), and (b): $\tilde{\beta} = 10$, $k = 0$ ($250 \leq t < 50$); $k = 0.135$ ($t \geq 100$). Dashed lines at $t = 50$ and $t = 250$ indicate the times of turning the control on and off, respectively.

3.2 Coupled cell dynamics

Let us consider 1-D ring of diffusively coupled Chua's cells:

$$\begin{aligned} \dot{x}_i &= a(y_i - f(x_i)) + D(x_{i-1} - 2x_i + x_{i+1}) \\ \dot{y}_i &= x_i - y_i + z_i \\ \dot{z}_i &= -by_i \end{aligned} \quad (14)$$

where

$$f(x_i) = m_3 x_i + \frac{1}{2} \sum_{j=1}^3 ((m_{j-1} - m_j)(|x_i + c_j| - |x_i - c_j|)) ,$$

$i = 1, 2, \dots, n$; $x_0 \equiv x_{25}$, and $x_{26} \equiv x_1$. The values of parameters a , b , m_0 , m_1 , m_2 , m_3 , c_1 , c_2 , and c_3 are kept the same as in §3.1.

Substituting $y = -\dot{z}/b$ to the second equation of (14), obtain

$$\begin{aligned} \ddot{z}_i + \dot{z}_i + bz_i &= -bx_i \\ \dot{x}_i + af(x_i) &= D(x_{i-1} - 2x_i + x_{i+1}) - \frac{a}{b}\dot{z}_i \end{aligned} \quad (15)$$

Again, to apply the above approach, one need to add the feedback $g(\dot{z})$ to the first equation of system (15). Performing similar substitutions as in §3.1, obtain the control feedback to be applied to second equation of the system (14):

$$g(y_i) = -k_i \tanh(\tilde{\beta}y_i). \quad (16)$$

Because of all cells are identical and the diffusion is homogeneous, take $k_i \equiv k, i = 1, 2, \dots, n$.

Consider the ring of 25 cells. Again, choose the initial conditions in the basin of the pathological state. Unlike previous case of the single cell, high amplitude oscillations of the cell ring are modulated by a lower frequency. Comparing systems (12) and (15), conclude that this modulation is caused by coupling of the oscillator (15) through its dynamical feedback (the variable x_i). Applying the same strategy as in §3.1, switch the trajectory to the attraction basin of the basal state (Fig. 2, $100 \leq t \leq 175$). In this case, the low amplitude period-1 cycle is stabilized, which is the induced attractor for the unperturbed system. Turning the perturbation off, keep the trajectory in the basin of physiological attractor and, at the same time, converts the induced (by the perturbation) attractor to the natural one (see Fig. 2 at $t \geq 175$).

As in the single cell case, further increasing k leads to stabilization of fixed point corresponding to the deep anesthesia state.

4 Discussion and conclusions

The approach utilizes simple feedback depending solely on the output signal and, hence, is especially useful when the system parameters are inaccessible or are costly to adjust. This case is characteristic of biological systems (see, for example, [6] where the occasional proportional feedback utilizing the accessible variable instead of the parameter is proposed for controlling the heart dynamics). The particular type of the perturbation is rather relative - most important, it should comply with the condition (5).

In this paper, we simply increase the feedback strength to adjust the oscillation energy to different levels. The above strategy does not require any computation of the control signal and, hence, is applicable for control as well as identification of unknown systems.

For the network dynamics, reducing the number of control inputs is an important issue. For global

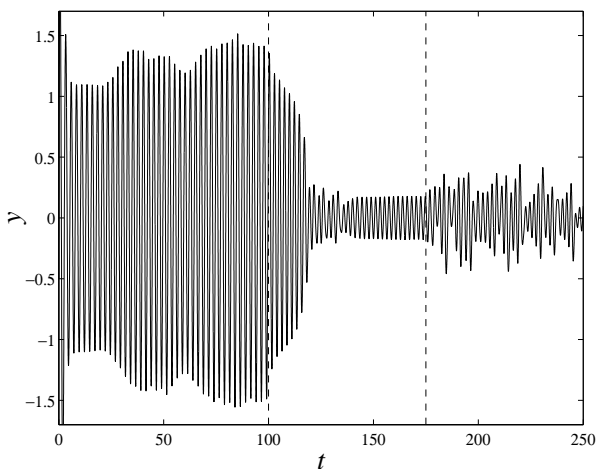


Figure 2: Dynamics of the state variable y of the system (14)-(16) at $\tilde{\beta} = 1$, and $k = 0$ ($175 \leq t < 100$); $k = 0.4$ ($t \geq 100$). Dashed lines at $t = 100$ and $t = 175$ indicate the times of turning the control on and off, respectively.

coupling, when any oscillator affects directly every other oscillator in the networks, controlling only a single oscillator is shown to suffice [5]. Similar results can be expected in locally connected networks at strong coupling. Unlike, at weak coupling, locally connected networks can be separated on clusters of different (say, chaotic and periodic) dynamics. At intermediate coupling, the cluster competition and transitional waves from one region to another can be expected. For the cortex dynamics, the later will mean competition of physiological and pathological regimes.

The considered models can qualitatively represent phenomena of another dynamical disease, the Parkinsonian tremor [7]. Indeed, the low amplitude chaotic attractor can account for the physiological tremor, whereas the high amplitude periodic states can be assigned to the pathological regimes.

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