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Biological Asymmetric and Parallel - Symmetric Neural Networks with Nonlinear Functions

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Abstract
In the biological visual neural networks, one of prominent features, is nonlinear functions, which play important roles in the visual systems. The visual information is inputted first to the retinal neural networks, then is transmitted and finally is processed in the visual cortex and middle temporal area of the primate brain. In these networks, it is reported that some nonlinear functions will process the visual information effectively. However, it is not clarified that what kinds of nonlinear functions will process what kinds of works in the visual processing. The order of the nonlinear functions is not yet clarified to show the mechanism of the visual information functions. In this paper, we discuss the nonlinear functions in the visual systems to clarify the structural and functional properties of the networks .

1. Introduction

The visual information is processed firstly in the retina, thalamus on the way and finally cortex , which are on the visual pathway of the biological networks. It is expected that what kinds of functions are realized in the structures of the networks. What are the relations between the structures and the functions in the neural networks? Are there any computational fundamental mechanisms in the retina and also the cortex in the visual neural networks? In this paper , first we investigate relations between the functions and structures in the retina. Next, we will discuss the relations between the functions and the structure in the visual area and the middle temporal area of the cortex. We speculate the common principle between the functions and the structures in these networks.

Retinal ganglion cells produce two types of response: linear and nonlinear responses. The nonlinear responses are generated by the separate and independent nonlinear pathway on the visual one, which is different from the linear pathway [3,9,10]. The nonlinear pathway is composed of a sandwich model in filters concatenation. These nonlinear characteristics, are studied in the perception of the Fourier and non-Fourier motion movement in the visual processing, which shows the responses to the moving objects.[3,7,8,13,17]

It is very useful and important to clarify the relation between the structure and the function of the linear and the nonlinear pathways in the biological neural network . In this study, the structural and functional relations, are discussed in the neural networks for the visual processing. The structure here of the neural network , is classified to the asymmetric network and the symmetric neural network. The function here of the neural network, is focused on the movement detection or stimulus changes. Asymmetric neural networks are shown in the biological neural network as the catfish retina [9,10]. Horizontal and bipolar cell responses are linearly related to the input modulation of light , while amacrine cells work linearly and nonlinearly in their responses [12]. These cells make asymmetrical neural networks in the retina. As the movement detection mechanism, several models have been proposed in the biological system [1,5,11,17]. To make clear the difference among asymmetrical networks , we applied non-linear analysis developed by Nobert Wiener [8].

First , it is shown that an asymmetric neural network is closely related to the nonlinear characteristics of the networks . Here, we define equations for the movement detection and the direction of the movement. It is shown that the asymmetric networks with the odd order nonlinearity on one pathway and the even order nonlinearity on the other pathway, satisfy both equations. Second, this fundamental relation holds in the parallel symmetric networks with half-wave rectification nonlinearity as shown in the networks of visual cortex

2. Asymmetric Neural Networks

Naka[9] presented a simplified, but essential networks of catfish inner retina., which has an asymmetric structure of the neural networks. It has already been established that the neurons in the biological retina, have a duplex receptive field organization. Such neurons have a receptive field center and a linear surround mechanism, but also receive input from another receptive field mechanism , the nonlinear subunits[3] In their study, it was clarified that the nonlinear subunits give rise to the characteristic even-order nonlinear responses. These findings suggest that these cells make asymmetric neural networks in the retina. . A biological network of catfish retina is shown in Fig.1[5,6,9] , which might process the spatial interactive information between bipolar cells **B1** and **B2** . The bipolar **B** cell response is linearly related to the input modulation of light . The **C** cell shows an amacrine cell, which plays an important roll in the nonlinear function as squaring of the output of the bipolar cell **B2**.

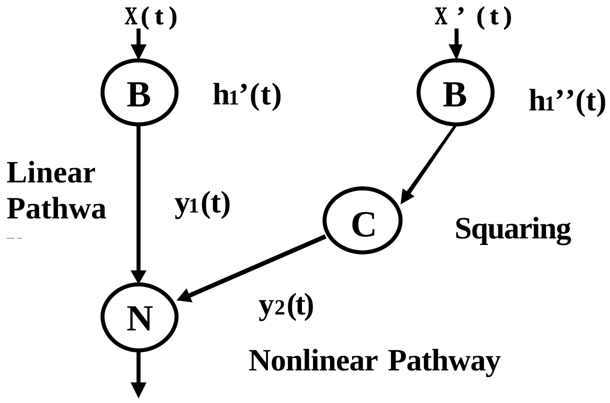

Fig . 1 Asymmetric Neural Network for Spatial Interaction

The **N** amacrine cell was clarified to be time-varying and differential with band-pass characteristics in the function. It is shown that N cell response is realized by a linear filter , which is composed of a differentiation filter followed by a low-pass filter. Thus the asymmetric network in Fig.1 is composed of a linear pathway and a nonlinear pathway. Barlow and Levick demonstrated that the asymmetric network with lateral inhibition is crucial in motion detection [2]. In this paper , we show that the nonlinearities and correlations are important factors to classify the asymmetric network and the symmetric network. In the following section , we derive the a – equation of the movement, which shows the quantity of the change of the moving stimulus. Further , we derive the equation of the movement direction. Thus, the first equation shows the detection of the moving stimulus , while the second equation shows the direction of the movement . These two equations means a movement vector : the quantity and the direction, which is a kind of the optical flow equation.

3. Asymmetric Neural Network with Quadratic Nonlinearity

Movement perception is carried out firstly in the retinal neural network. Asymmetric neural network in the catfish retina , has characteristic asymmetric structure with aquadratic nonlinearity as shown in Fig.1. Fig.2 shows a schematic diagram of a motion problem in front of the asymmetric network in Fig.1. The slashed light is assumed to move from the left side to the right side, gradually. For the simplification of the analysis of the spatial interaction, we assume here the input functions $x(t)$ and $x''(t)$ to be Gaussian white noise, whose mean values are zero, but their deviations are different in their values. In Fig.2, moving stimulus shows that $x(t)$ merges into $x''(t)$, thus $x''(t)$ is mixed with $x(t)$. Then , we indicate the right stimulus by $x'(t)$. By introducing a mixed ratio , a , the input function of the right stimulus , is described in the following equation , where $0 \leq a \leq 1$ and $ß = 1 - a$

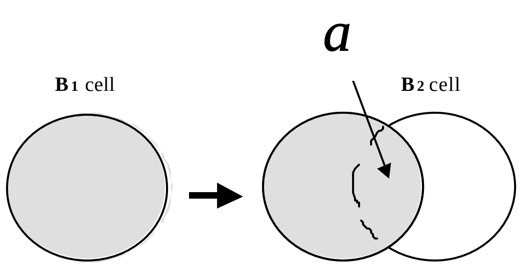

Fig . 2 Schematic Diagram of Motion Problem for Spatial Interaction hold. Fig.2 shows that the moving stimulus is described in the following equation,

$$x'(t) = ax(t) + ßx''(t) \qquad (1)$$

Let the power spectrums of $x(t)$ and $x''(t)$, be p and p',respectively and an equation $p = kp''$ holds for the coefficient k, because we assumed here that the deviations of the input functions are different in their values. Fig.2 shows that the slashed light is moving from the receptive field of **B1** cell to the field of the **B2** cell . The mixed ratio of the input $x(t)$, a is shown in the receptive field of **B2** cell.

First , on the linear pathway of the asymmetrical network in Fig.1, the input function is $x(t)$ and the output function is $y(t)$,where

$$y(t) = y_1(t) + y_2(t) \qquad (2)$$

We can compute the 0-th order Wiener kernel C_0 , the 1-st order one $C_{11}(?)$,and the 2-nd order one $C_{21}(?_1, ?_2)$ on the linear pathway by the cross-correlations between $x(t)$ and $y(t)$. The suffix i , j of the kernel $Cij(\bullet)$, shows that i is the order of the kernel and $j = 1$ means the linear pathway, while $j = 2$ means the nonlinear pathway. Then , the 0-th order kernel under the condition of the spatial interaction of cell's impulse response functions $h_1?(t)$ and $h_1?(t)$, becomes

$$C_0 = E[y(t)]$$
$$= p(a^2 + kß^2)\int_0^\infty (h_1?(t_1))^2 dt_1 \qquad (3)$$

The 1 -st order kernel is derived as follows ,

$$C_{11}(?) = \frac{1}{p} E[y(t)x(t-?)] = h_1'(?) \qquad (4)$$

, since the last term of the second equation becomes zero. The 2-nd order kernel becomes ,

$$C_{21}(?_1, ?_2) = \frac{1}{2p^2} E[(y(t)-C_0)(x(t-?_1)x(t-?_2)]$$
$$= a^2 h_1?(?_1)h_1?(?_2) \qquad (5)$$

From equations (1) , (4) and (5) , the ratio a is a mixed coefficient of $x(t)$ to $x'(t)$, is shown by a^2 as the amplitude of the second order Wiener kernel. Second , on the nonlinear pathway , we can compute the 0-th order kernel C_0 , the first order kernel $C_{12}(?)$ and the 2 -nd order kernel $C_{22}(?_1, ?_2)$ by the cross-correlations between $x(t)$ and $y(t)$ as shown in the following.

$$C_{12}(?) = \frac{1}{p(a^2 + kß^2)} E[y(t)x'(t-?)]$$
$$= \frac{a}{a^2 + k(1-a)^2} h_1'(?) \qquad (6)$$

and

$$C_{22}(?_1, ?_2) = h_1?(?_1)h_1?(?_2) \qquad (7)$$

The motion problem is how to detect the movement in the increase of the ratio a in Fig.2. This implies that for the motion of the light from the left side circle to the right one, the ratio a can be derived from the kernels described in the above, in which the second order kernels C_{21} and C_{22} are abbreviated in the representation of equations (5) and (7).

$$(C_{21}/C_{22}) = a^2 \qquad (8)$$

holds. Then , from (8) the ratio a is shown as follows .

$$a = \sqrt{\frac{C_{21}}{C_{22}}} \qquad (9)$$

The equation (9) is called here a - equation , which implies the change of the movement stimulus on the network and shows the detection of the movement change by the a . This shows that the a - equation is determined by the second order kernels on the linear pathway and the nonlinear one in the network . Thus, the second order nonlinearity plays an important role in the detection of the movement. In this paper, we apply the Wiener nonlinear analysis to these networks.
The Wiener analysis is based on the cross-correlations. The higher order correlations are useful in their solutions. In the biological systems, Reichardt [11] shows the usefulness in the perception of the their system. Here, we show that the asymmetrical network of the structure and the correlation functions are important factor in the movement perception. The equation (9) does not show the direction of the movement. Any measure to show the direction of the movement, is needed.

So, we will discuss how to detect the direction of the movement.

From the first order kernels C_{11} and C_{12} , and the second order kernels in the above discussion ,which are abbreviated in the time function, the following movement equation holds,

$$\frac{C_{12}}{C_{11}} = \frac{\sqrt{\frac{C_{21}}{C_{22}}}}{\frac{C_{21}}{C_{22}} + k\left(1 - \sqrt{\frac{C_{21}}{C_{22}}}\right)^2} \tag{10}$$

, where k in the equation shows the difference of the power of
the stimulus between the receptive fields of the left and the right cells B_1 and B_2.

$$C_{12}(?) = ßh_1´(?) \tag{13}$$

$$C_{22}(?_1, ?_2) = h_1?(?_1)h_1?(?_2) \tag{14}$$

From the movement of the stimulus from the left to the right, the equation (10) is derived. Here, we have a problem whether the equation (10) is different from the equation derived , under the condition that the movement of the stimulus from the right to the left. If the equation (10) from the left to the right, is different from the equation from the right to the left, these equations will suggest different movement directions, respectively as the vectors.

Then, from the equation (9) , the differentiation of the equation (9) by the time, will show the velocity of the movement

Here, we will consider the case of the movement of the stimulus from the right to the left.

In the opposite direction from the right to left side stimulus, the schematic diagram of the stimulus movement, is shown in Fig.3. The left side stimulus moves from the receptive field of the cell B_1 to the receptive field of the right cell B_2 , gradually.

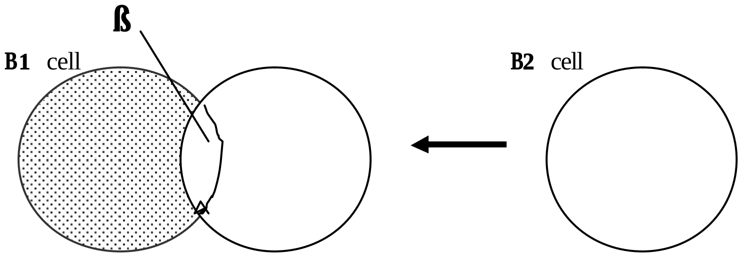

Fig.3 Schematic Diagram of the Stimulus Movement from the Right to the Left

In Fig.3 , the movement of the stimulus is shifted from the right to the left. Then, the following equations are derived on the linear pathway in Fig.1.

$$C_{11}(?) = h_1´(?) \tag{11}$$

$$C_{21}(?_1, ?_2) = \frac{k^2ß^2}{(a^2 + kß^2)^2} h_1?(?_1)h_1?(?_2) \tag{12}$$

Similarly, the following equations are derived on the nonlinear pathway,

From equations (11) and (13), the ratio ß is derived, which is abbreviated in the notation.

$$ß = \frac{C_{12}}{C_{11}} \tag{15}$$

, where the ß is the parameter which shows
the ratio of the right stimulus, introduced to the
left side receptive field of the cell B_1.

The differentiation of the parameter ß by the time, shows the velocity of the movement of the right side stimulus.

In the equations (11), (12), (13) and (14), the impulse response functions, are unknown parameters of the cells B_1 and B_2. By the elimination of the these response functions from these equations, the following equation is derived.

$$\frac{C_{11}}{C_{12}} = \frac{k\sqrt{\frac{C_{21}}{C_{22}}}}{\left(1 - \frac{C_{12}}{C_{11}}\right)^2 + k\left(\frac{C_{12}}{C_{11}}\right)^2} \qquad (16)$$

Equations (10) and (16) are transformed to the following equations (17) and (18), respectively.

$$a^3(k+1) - a^2(3k+1) + a(3k+1) - k = 0 \qquad (17)$$

$$a^3(k+1) - 2a^2k + 2ka - k = 0 \qquad (18)$$

To satisfy the equality between equations (17) and (18), the following equation must be satisfied.

$$a^2(k+1) - ak - a = 0 \qquad (19)$$

But, we have no a solutions in the equation (19) under the condition $0 < a < 1$ and $k>0$. This implies that equations (10) and (16), are different in their directional values for the stimulus movement. Thus, the equation (10) characterizes the preferred direction from the left to the right, while the equation (16) characterizes the null direction from the right to the left. Further, the differentiation of a - equations (9) and (15), shows the velocity of the moving stimulus. Thus, equations (9) and (10) (similarly (15) and (16)) show a representation of the optical flow of the moving stimulus. The asymmetric neural network in Fig.1, which is characterized by equations (10) and (16) for the moving stimulus, consists of the linear pathway (1-st order nonlinearity: odd order) and the nonlinear pathway (2-nd order nonlinearity: even order).

Those network characteristics are generalized as follows: the asymmetric network with the odd order nonlinearity on the one pathway and the even order nonlinearity on the other pathway, has both a - equation (9)(or (15))and the directional equation (10)(or (16)) on the 1-dimensional coordinate.

Based on the above analysis in the asymmetric nonlinear networks, the parallel processing in the brain cortex network, is discussed as follows. The visual cortex areas, V1 and MT, are studied to have the role of the tracking of the moving stimulus[15,16]. The nonlinearity in the visual system, has a problem of the it's order [13]. Heeger and Simoncelli presented a parallelization network model with half-wave rectification nonlinearity in V1 and MT cortex area in the brain[14,15,16]. In this paper, this parallel processing model is interpreted analytically to have the tracking function of the moving stimulus, which is based on the asymmetrical nonlinear networks described in the above.

In this paper , we will discuss the problem of the higher-order nonlinearities from the view of the movement detection . We assume here an asymmetric neural network with the third-order nonlinearity as shown in Fig . 3 .Fig.3 is not seen in the biological network , which is assumed here for the analysis of the nonlinear asymmetrical network.
The linear pathway on the left side, is equivalent to the 1-st order nonlinear pathway. The nonlinear pathway on the right side, has the the 3-rd order nonlinearity. Thus, this asymmetrical network composed of the odd and the odd order nonlinearities. On the linear pathway in Fig . 3, the following kernels are derived.

$$C_{12}(?_1, ?_2) = 0$$
$$C_{13}(?_1, ?_2, ?_3) = a^3 h_1?(?_1) h_1?(?_2) h_1?(?_3) \qquad (20)$$

On the right pathway in Fig.3 (tripling of **T** cell) , the kernel becomes,

$$C_{22}(?_1, ?_2) = 0 \qquad (21)$$

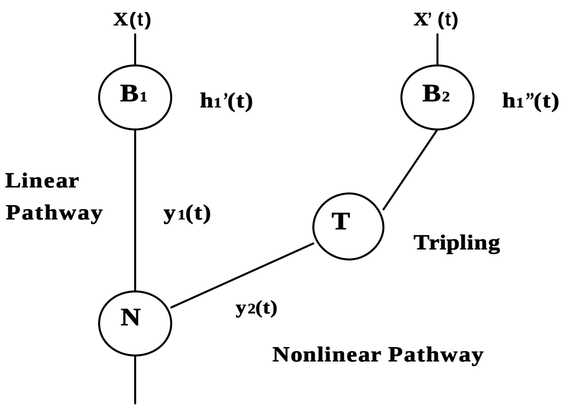

Fig . 4 Asymmetric Network with 3-rd Order Nonlinearity

From the equations (20) and (21) , the a - equation (22) is obtained as follows ,

$$a = \left\{ \frac{C_{13}(?_1 , ?_2 , ?_3)}{C_{23}(?_1 , ?_2 , ?_3)} \right\}^{\frac{1}{3}} \tag{22}$$

4. Symmetric Neural Network with Half–Wave Rectification Nonlinearity

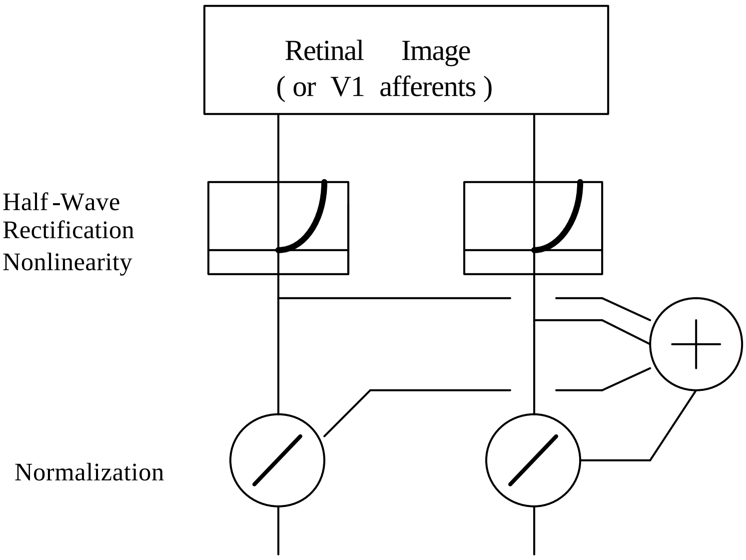

Fig . 4 Symmetric Network with Half – Wave Rectification Nonlinearity (Simoncelli and Heeger, et al) Modeled in Cortical Area V1 and Middle Temporal Area MT.

In the cortical area V1 and the middle temporal area MT, the movement detection is carried out. These areas models shown in Fig .4 (Simoncelli et al). We call here the symmetric network with the half-wave rectification nonlinearity. In Fig . 4, the half-wave rectification followed by the normalization , is approximated by the nonlinear function as a sigmoid function in the following,

$$f(x) = \frac{1}{1 + e^{-?(x-?)}} \tag{23}$$

By Taylor expansion of the equation (23) at $x = ?$, the equation (24) is derived as shown in the followings,

$$\begin{aligned} f(x)_{x=?} &= f(?) + f\,?(?)(x-?) + \frac{1}{2!} f\,?(?)(x-?)^2 + \cdots \\ &= \frac{1}{2} + \frac{?}{4}(x-?) + \frac{1}{2!}\left(-\frac{?^2}{4} + \frac{?^2 e^{-??}}{2}\right)(x-?)^2 + \cdots \end{aligned} \tag{24}$$

Equation (22) shows the asymmetric network can detect the change of the stimulus as the a - equation, while the movement equation does not exist as the (10) and (16), which show the direction of the movement , respectively.

It is shown that the asymmetric networks with the odd nonlinearity on the left side pathway and another odd nonlinearity on the right side pathway cannot detect the direction of the movement. Similarly, it is shown the asymmetric networks with the even order on the one side pathway and with another even order on the other side pathway, cannot detect the direction of the movement.

Simoncelli and Heeger presented a model of the cortical area V1 and the middle temporal area MT, which detect the movement stimulus in the vertebrate brain[16]. Lu and Sperling showed the human visual motion perception model, in which the half – wave rectification plays an significant role in their mechanisms[17].

In the equation (24) , a sigmoid function is adopted as the approximation of the half-wave squaring nonlinearity, which is followed by the normalization operation as shown in Fig. 4. In the Taylor expansion of the sigmoid function, the parameter $?$ becomes to be large as $? \geq 8$ and $? \square 0.5$ by max of $f(x) = 1$ in the equation (14) . Thus , the first order $(x - ?)$ term $? / 4$ exists and the second order term $(x-?)^2$ also exists by the relation $?? \square \log_e 2$. From the section 3 , both the detection and the direction of the movement stimulus , is fundamentally realized in the asymmetric network with the linear pathway (equivalent to the 1-st order nonlinearity) and the 2-nd order nonlinear pathway. This relation is expanded in the asymmetric networks with the odd order nonlinearity pathway and the even order nonlinearity. From the sigmoid function applied In Fig . 4 , the half-wave rectification nonlinearity , includes the 1-st order nonlinearity, the 2-nd order nonlinearity, also the 3-rd order and higher order nonlinearities. Fig . 4 shows a parallel - symmetric network , which is different from the asymmetric network as shown in Fig . 1. When we pick up two parallel pathways with half-wave rectification , the combination of the odd order nonlinearity on the left side pathway and the even order nonlinearity on the right side pathway, can detect the movement and the direction of the stimulus and viceversa in the nonlinearities on the pathways. It is proved that the a-equation and the movement equation are not derived in the symmetric network with the even order nonlinearity on the pathway and the other even nonlinearity on the another pathway. Thus, both the even nonlinearities on the parallel pathways, do not have any works on the correlation processing. The even and the odd nonlinearities together , play an important role in the movement correlation processing.

5. Conclusion

It is important to study the biological neural networks from the view point of their structures and functions. In this paper , their structures are classified to two types of the networks : asymmetric network and symmetric network. The asymmetric network is based on the biological neural network of the catfish retina , while the symmetric network is based on the visual cortex and middle temporal area. First, it is shown that the asymmetric networks with nonlinear quadratic characteristic, satisfy both the moving detection equation (a -equation) and the movement direction equation. Further , the asymmetric network with the even and the odd nonlinearities, has powerful ability in the information processing. Based on the asymmetric network principle, it is shown that the parallel-symmetric network with half-wave rectification , also has both movement equations described in the above.

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