

A Binary Neural Decision Table Classifier

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Abstract

In this paper, we introduce a neural network-based decision table algorithm. We focus on the implementation details of the decision table algorithm when it is constructed using the neural network. Decision tables are simple supervised classifiers which, Kohavi demonstrated, can outperform state-of-the-art classifiers such as C4.5. We couple this power with the efficiency and flexibility of a binary associative-memory neural network. We demonstrate how the binary associative-memory neural network can form the decision table index to map between attribute values and data records. We also show how two attribute selection algorithms, which may be used to pre-select the attributes for the decision table, can easily be implemented within the binary associative-memory neural framework. The first attribute selector uses mutual information between attributes and classes to select the attributes that classify best. The second attribute selector uses a probabilistic approach to evaluate randomly selected attribute subsets.

Introduction

Supervised classifier algorithms aim to predict the class of an unseen data item. They induce a hypothesis using the training data to map inputs onto classified outputs (decisions). This hypothesis should then correctly classify previously unseen data items. There is a wide variety of classifiers including: decision trees, neural networks, Bayesian classifiers, Support Vector Machines and k-nearest neighbour.

We have previously developed a k-NN classifier[HA04] using an associative memory neural network called the Advanced Uncertain Reasoning Architecture (AURA)[A95]. In this paper, we extend the approach to encompass a decision table supervised classifier, coupling the classification power of the decision table with the speed and storage efficiency of an associative memory neural network

The decision table has two components: a schema and a body. The schema is the set of attributes pre-selected to represent the data and is usually a subset of the data's

total attributes. There are various approaches for attribute selection; we discuss two later in this paper. The body is essentially a table of labelled data items where the attributes specified by the schema form the rows and the decisions (classifications) form the columns. Each column is mutually exclusive and represents an equivalence set of records as defined by the attributes of the schema. Kohavi [K95] uses a Decision Table Majority (DTM) for classification whereby if an unseen item exactly matches a stored item in the body then the decision table assigns the stored item's decision to the unseen item. However, if there is no exact match then the decision table assigns the majority class across all items to the unseen item. Our decision table approach implements both DTM and proximity-based matching as implemented in our k-NN classifier whereby if there is no exact match then the decision table assigns the class of the nearest stored item to the unseen item.

RAM-based Neural Networks

The AURA C++ library provides a range of classes and methods for rapid partial matching of large data sets [A95]. In this paper we define partial matching as the retrieval of those stored records that match some or all of the input record. In our AURA decision table, we use best partial matching to retrieve the records that are the top matches.

AURA belongs to a class of neural networks called Random Access Memory (RAM-based) networks. RAM-based networks were first developed by Bledsoe & Browning [BB59] and Aleksander & Albrow [AA68] for pattern recognition and led to the WISARD pattern recognition machine [ATB84]. See also [A98] for a detailed compilation of RAM methods.

RAMs are founded on the twin principles of matrices (usually called Correlation Matrix Memories (CMMs)) and n-tupling. Each matrix accepts m inputs as a vector or tuple addressing m rows and n outputs as a vector addressing n columns of the matrix. During the training phase, the matrix weights \mathbf{M}^k are incremented if both the input row \mathbf{I}_j^1 and output column \mathbf{O}_j^k are set.

Therefore, training is a single epoch process with one training step for each input-output association preserving the high speed. During recall, the presentation of vector \mathbf{I}_j elicits the recall of vector \mathbf{O}_j as vector \mathbf{I}_j contains all of the addressing information necessary to access and retrieve vector \mathbf{O}_j . This training and recall makes RAMs computationally simple and transparent with well-understood properties. RAMs are also able to partially match records during retrieval. Therefore, they can rapidly match records that are close to the input but do not match exactly.

AURA

AURA has been used in an information retrieval system[H01], high speed rule matching systems[AKL95], 3-D structure matching[TA00] and trademark searching[AA98]. AURA techniques have demonstrated superior performance with respect to speed compared to conventional data indexing approaches [HA01] such as hashing and inverted file lists which may be used for a decision table body. AURA trains 20 times faster than an inverted file list and 16 times faster than a hashing algorithm. It is up to 24 times faster than the inverted file list for recall and up to 14 times faster than the hashing algorithm. AURA techniques have demonstrated superior speed and accuracy compared to conventional neural classifiers [ZAK99].

The rapid training, computational simplicity, network transparency and partial match capability of RAM networks coupled with our robust quantisation and encoding method to map numeric attributes from the data set onto binary vectors for training and recall make AURA ideal to use as the basis of an efficient implementation. A more formal definition of AURA, its components and methods now follows.

Correlation Matrix Memories (CMMs) are the building blocks for AURA systems. AURA uses binary input \mathbf{I} and output \mathbf{O} vectors to train records in to the CMM and recall sets of matching records from the CMM as in Equation 1 and Figure 1.

Equation 1

$$\text{CMM} = \bigvee_{\text{all } j} \mathbf{I}_j \times \mathbf{O}_j^T \text{ where } \bigvee \text{ is logical OR}$$

Training is a single epoch process with one training step for each input-output association (each $\mathbf{I}_j \times \mathbf{O}_j^T$ in Equation 1) which equates to one step for each record in the data set.

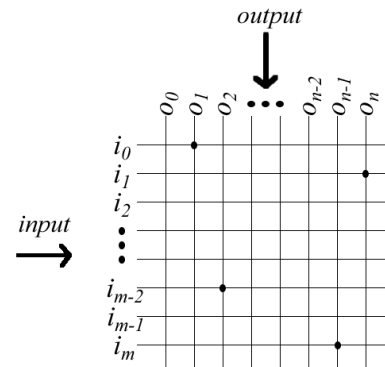


Figure 1 Showing a CMM with input vector \mathbf{i} and output vector \mathbf{o} . Four matrix locations are set following training $i_0o_0, i_2o_{n-2}, i_{m-1}o_0$ and $i_m o_n$.

For the methodology described in this paper, we:

- Train the data set into the CMM (decision table body CMM) which indexes all records in the data set and allows them to be matched.
- Select the attributes for the schema using schema CMMs. We describe two selection algorithms. One uses a single CMM but the second algorithm uses two coupled CMMs.
- Match and classify unseen items using the trained decision table.

Data

For the data sets:

- Symbolic and numerical unordered attributes are enumerated and each separate token maps onto an integer (*Text ? Integer*) which identifies the bit to set within the vector. For example, a *SEX_TYPE* attribute would map as, (*F ? 0*) and (*M ? 1*).

Kohavi’s DTM methodology is principally aimed at symbolic attributes but the AURA decision table can handle continuous numeric attributes.

- Any real-valued or ordered numeric attributes, are quantised (mapped to discrete bins) and each individual bin maps onto an integer which identifies the bit to set in the input vector.

A range of input values for attribute f map onto each bin which in turn maps to a unique integer to index the vector as in Equation 2. The range of attribute values mapping to each bin is equal.

Equation 2

$$\mathfrak{R}_{f_i} \rightarrow \text{bins } f_k \mapsto \text{Integer } f_k + \text{offset } f \text{ where } i \in FV_f \wedge \text{cardinality}(\text{Integer } f) \equiv \text{cardinality}(\text{bins } f)$$

In Equation 2, $offset_f$ is a cumulative integer offset within the binary vector for each attribute f and $offset_{f+1} = offset_f + nBins_f$, where $nBins_f$ is the number of bins for attribute f , FV_f is the set of attribute values for attribute f , $?$ is a many-to-one mapping and \mapsto is a one-to-one mapping.

This quantisation (binning) approach aims to subdivide the attributes uniformly across the range of each attribute. The range of values is divided into b bins such that each bin is of equal width. The even widths of the bins prevent distortion of the inter-bin distances.

Once the bins and integer mappings have been determined, we map the records onto binary vectors. Each attribute maps onto a consecutive section of bits in the binary vector.

```

For each record in the data set
  For each attribute
    Calculate bin for attribute value;
    Set bit in vector as in Equation 2;

```

Each binary vector represents a record from the data set

Body Training

The decision table body is an index of all contingencies and the decision to take for each. Input vectors represent quantised records and form an input I_j to the CMM during training. The CMM associates the input with a unique output vector O_j^T during training which represents an equivalence set of records. This produces a CMM where the rows represent the attributes and their respective values and the columns represent equivalence sets of records (where equivalence is determined by the attributes designated by the schema). We use an array of linked lists to store the equivalence sets of records and a second array to store the counts of each class for the equivalence set as a histogram. The algorithm is:

- 1) Input vector to CMM;
- 2) Threshold at value nF ;
- 3) If exact match
- 4) Add the record to column list;
- 5) Add class to histogram;
- 6) Else train record as next column;

nF is the number of attributes. Steps 1 and 2 are equivalent to testing for an exact match during body recall as described next. Figure 3 shows a trained CMM where each row is an attribute value and each column represents an equivalence set.

Body Recall

The decision table classifies by finding the set of matching records. To recall the matches for a query record, we firstly produce an input vector by quantising

the target values for each attribute to identify the bins and thus CMM rows to activate as in Equation 2. To retrieve the matching records for a particular record, AURA effectively calculates the dot product of the input vector I_k and the CMM, computing a positive integer-valued output vector O_k (the summed output vector) as in Equation 3 and Figure 2 & Figure 3.

$$\text{Equation 3 } O_k^T = I_k \bullet \text{CMM}$$

The AURA technique thresholds the summed output O_k to produce a binary output vector as in Figure 2 for exact match and Figure 3 for a partial match.

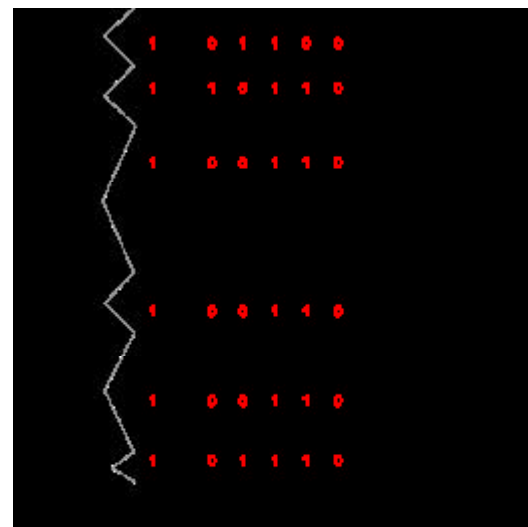


Figure 2 Diagram showing the CMM recall for an exact match. The left hand column is the input vector. The dot is the value for each attribute (a value for an unordered attribute or a bin for an ordered numeric attribute). AURA multiplies the input vector by the values in the matrix columns, using the dot product, sums each column to produce the summed output vector and then thresholds this vector at a value equivalent to the number of attributes in the input (6 here) to produce the thresholded attribute vector which indicates the matching column (the middle column here).

For exact match (as in Kohavi's DTM), we use the Willshaw threshold. It sets a bit in the thresholded output vector for every location in the summed output vector that has a value higher than a threshold value. The threshold value is set to the number of attributes nF for an exact match. If there is an exact match there will be a bit set in the thresholded output vector indicating the matching equivalence set. It is then simply a case of looking up the class histogram for this equivalence set in the stored array and classifying the record by the majority class in the histogram. If there

are no bits set in the thresholded output vector then we classify the unseen record according to the majority class across the data set.

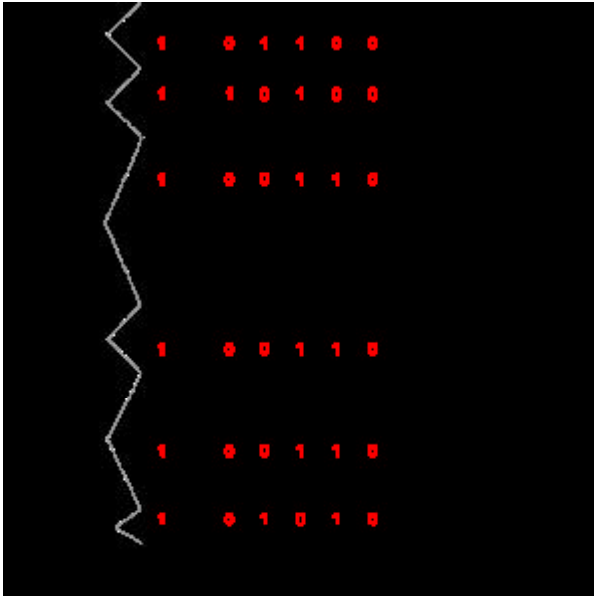


Figure 3 Diagram showing the CMM recall for a partial match. The left hand column is the input vector. The dot is the value for each attribute (a value for an unordered attribute or a bin for an ordered numeric attribute). AURA multiplies the input vector by the values in the matrix columns, using the dot product, sums each column to produce the summed output vector and then thresholds this vector at a value equivalent to the highest value in the vector (5 here) to produce the thresholded attribute vector which indicates the matching column (the middle column here).

For partial matching, we use the LMax threshold. LMax thresholding essentially retrieves *at least L* top matches. It sets a bit in the thresholded output vector for every location in the summed output vector that has a value higher than a threshold value. The AURA C++ library automatically sets the threshold value to the highest integer value that will retrieve at least *L* matches. For the AURA decision table, *L* is set to the value of **1**. There will be a bit set in the thresholded output vector indicating the best matching equivalence set. It is then simply a case of looking up the class histogram for this equivalence set in the stored array and classifying the unseen record as the majority class. We note there may be more than one best matching equivalence set so the majority class across all best matching sets will need to be calculated.

Schema Training

In the decision table body CMM, the rows represented attribute values and the columns represented equivalence sets. In the schema CMM used for the first attribute selection algorithm, the rows represent attribute values and the columns represent individual records. For our second attribute selection algorithm, we use two CMMs where the first CMM indexes the second CMM. In the first CMM₁, the rows represent records and the columns represent attribute values. In the second CMM₂, the rows represent attribute values and the columns represent the records. This second CMM₂ is therefore identical to the CMM used for the first attribute selection algorithm

During training for the first attribute selection algorithm and CMM₂ of the second attribute selection algorithm, the input vectors \mathbf{I}_j represent the attribute values in the data records. The CMM associates the input with a unique output vector \mathbf{O}_j^T . Each output vector is orthogonal with a single bit set corresponding to the record's position in the data set, the first record has the first bit set in the output vector, the second and so on. During training for CMM₁, the records represent the input vectors \mathbf{I}_j with a single bit set and the output vectors \mathbf{O}_j^T represent the attribute values in the data records. The CMM training process is given in Equation 1.

Schema Attribute Selection

As with Kohavi, we assume that all records are to be used in the body and during attribute selection.

There are two fundamental approaches to attribute selection which are used in classification: a *filter* approach that selects the optimal set of attributes independently of the classifier algorithm and the *wrapper* approach that selects attributes to optimise classification using the algorithm. We examine two filter approaches which are more flexible than wrapper approaches as they are not directly coupled to the classification algorithm.

For a data set with N attributes there are $O(N^M)$ possible combinations of M attributes which is intractable to search exhaustively. In the following: we use one filter approach (**mutual information attribute selection**) that examines attributes on an individual basis and another probabilistic filter approach (**probabilistic Las Vegas algorithm**) that examines randomly selected subsets of attributes.

Mutual Information Attribute Selection

Wettscherek [W94] describes a **mutual information attribute selection algorithm** which calculates the

mutual information between class C and each attribute F_j . The mutual information between two attributes is “the reduction in uncertainty concerning the possible values of one attribute that is obtained when the value of the other attribute is determined”.

For unordered attributes, nFV is the number of distinct attribute values (f_j) for attribute F_j and $nClasses$ the number of classes (C):

Equation 4

$$I(C, F_j) = \sum_{i=1}^{nFV} \sum_{c=1}^{nClasses} p(C=c \wedge F_j=f_i) \cdot \log \frac{p(C=c \wedge F_j=f_i)}{p(C=c) \cdot p(F_j=f_i)}$$

For ordered numeric attributes, the technique computes the mutual information between a discrete random variable (class) and a continuous random variable (attribute). It estimates the probability function of the attributes using density estimation. We assume attribute F_j has density $f(x)$ and the joint density of C and F_j is $f(x, y)$.

Then the mutual information is:

Equation 5

$$I(C, F_j) = \int_x \sum_{c=1}^{nClasses} f(x, C=c) \cdot \log \frac{f(x, C=c)}{f(x) \cdot p(C=c)} dx$$

Equation 5 requires an estimate of the density function $f(x)$ and the joint density function $f(x, C=c)$. To approximate $f(x)$ and $f(x, C=c)$, we utilise the binning to represent the density which is analogous to the Trapezium Rule for using the areas of slices (trapezia) to represent the area under the graph for integration. We use the bins to represent strips of the probability density function and count the number of records mapping into each bin to estimate the density.

In AURA, for unordered data, the mutual information is given by Equation 6:

Equation 6

$$I(C, F_j) = \sum_{i=1}^{nRowsFV} \sum_{c=1}^{nClasses} \frac{nRowf_i}{n} \cdot \frac{n(BVf_i \wedge BVc)}{nRowf_i} \cdot \log \left(\frac{\frac{nRowf_i}{n} \cdot \frac{n(BVf_i \wedge BVc)}{nRowf_i}}{\frac{nClass_c}{n} \cdot \frac{nRowf_i}{n}} \right)$$

Where $nRowsFV$ is the number of rows in the CMM for attribute F_j , n is the number of records in the data set,

BVf_i is a binary vector (CMM row) for f_i , BVc is a binary vector with one bit set for each record in class c , $n(BVf_i \wedge BVc)$ is a count of the set bits when BVc is logically ANDed with BVf_i and $nClass_c$ is the number of records in class c .

In AURA, for real/discrete ordered numeric attributes, the mutual information is given by Equation 7:

Equation 7

$$I(C, F_j) = \sum_{i=1}^{nRowsFV} \sum_{c=1}^{nClasses} \frac{nRowb_i}{n} \cdot \frac{n(BVb_i \wedge BVc)}{nRowb_i} \cdot \log \left(\frac{\frac{nRowb_i}{n} \cdot \frac{n(BVb_i \wedge BVc)}{nRowb_i}}{\frac{nClass_c}{n} \cdot \frac{nRowb_i}{n}} \right)$$

Where nB is the number of bins in the CMM for attribute F_j , n is the number of records in the data set, BVb_i is a binary vector (CMM row) for f_i , BVc is a binary vector with one bit set for each record in class c , $n(BVf_i \wedge BVc)$ is a count of the set bits when BVc is logically ANDed with BVb_i and $nClass_c$ is the number of records in class c .

The technique assumes independence of attributes and ignores missing values. It is also the user’s prerogative to determine the number of attributes to select.

Probabilistic Las Vegas Algorithm

Liu & Setiono [LS96] introduced a **probabilistic Las Vegas algorithm** which uses random search and inconsistency to evaluate attribute subsets. For each equivalence set of records (where the records match according to the attributes designated in the schema), consistency is defined as the number of matching records minus the largest number of matching records in any one class. The inconsistency scores are summed across all equivalence sets to produce an inconsistency score for the particular attribute selection.

The technique uses random search to select attributes as random search is less susceptible to local minima than heuristic searches such as *forward* search or *backward* search. Forward search works by greedily adding attributes to a subset of selected attributes until some termination condition is met whereby adding new attributes to the subset does not increase the discriminatory power of the subset above a pre-specified threshold value. Backward search works by greedily removing attributes from an initial set of all attributes until some termination condition is met

whereby removing an attribute from the subset decreases the discriminatory power of the subset above a pre-specified threshold. A poor attribute choice at the beginning of a forward or backward search will adversely effect the final selection whereas a random search will not rely on any initial choices.

Liu and Setiono defined their algorithm as:

```

1)  Fbest = N;
2)  For j = 1 to MAX_TRIES
3)    S = randomAttributeSet(seed);
4)    nF = numberOfAttributes(S);
5)    If (nF < nFbest)
6)      If (InconCheck(S,D) < γ)
7)        Sbest = S; nFbest = nF;
8)  End for

```

Where D is the dataset, N the number of attributes and γ the permissible inconsistency score. Liu & Setiono recommend setting MAX_TRIES to $77 \times N^5$.

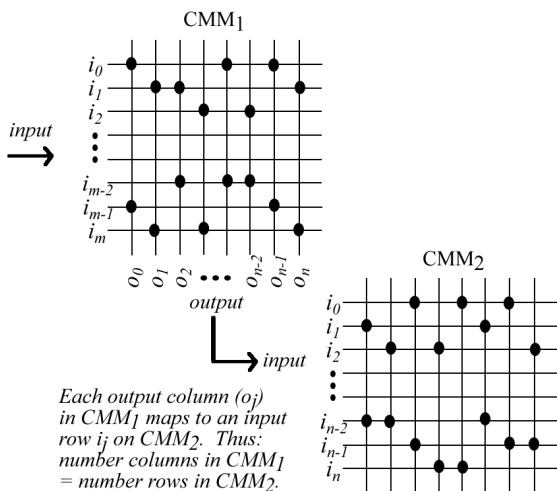


Figure 4 Showing the two CMM combination we use for Liu & Setiono’s algorithm. In the first CMM (CMM₁), the records index the rows (one row per record) and the attribute values index the columns. The outputs from the CMM (matching attribute values) feed straight into the second CMM(CMM₂), where the attribute values index the rows and the records index the columns (one column per record).

Liu and Setiono’s algorithm may be calculated simply using the AURA schema CMMs. We need to use two linked CMMs for the calculation as in Figure 4. We rotate the schema CMM (CMM₁) through 90°. CMM₁’s rows index the records and CMM₁’s columns index the attribute values. If we feed the outputs from CMM₁ (the activated attribute values) into CMM₂ then we can calculate the inconsistency scores easily. Line 6 of Liu and Setiono’s algorithm listed above then becomes:

```

Place all records in a queue Q;
While !empty(Q)
  Remove R the head record from Q;
  Activate row R in CMM1;
  Threshold CMM1 at value 1;
  Feed CMM1 output into CMM2;
  Threshold CMM2 at value Fbest;
  B = bits set in thresholded_vector;
  Max = cardinality of largest class;
  InconCheck(S,D) += B-Max;
End while

```

The queue effectively holds the unprocessed records. By activating the head record’s row in CMM₁ and Willshaw thresholding at value 1 (denoting all active columns (i.e., all attribute values in the record)), we can determine that records attribute values. When these values are fed into CMM₂, we effectively activate all records matching these values. This approach is the most efficient as the CMMs store *all* attributes and their values but we only focus on those attributes under investigation during each iteration of the algorithm. An alternative approach would be to just store those attributes selected in the random subset each time we execute line 6 of Liu and Setiono’s algorithm but the CMMs would need to be retrained many times (up to $77 \times N^5$).

After thresholding CMM₂ at the value F_{best} (the number of attributes), we retrieve the equivalence set of matching records where equivalence is specified by the current attribute selection in the algorithm $\{S\}$. It is then simply a matter of counting the number of matching records (the number of bits set in the thresholded output vector), calculating the number of these matching records in each class, identifying the largest class membership and subtracting the largest class membership from the number of records.

The algorithm has now processed all of the records in this equivalence set so it removes these records from the queue. If we repeat this process for each record at the head of the queue until the queue is empty, we will have processed all equivalence sets. We can then calculate InconCheck(S,D) for this attribute selection and compare it with the threshold value as in line 6 of Liu and Setiono’s algorithm.

Once we have iterated through Liu and Setiono’s algorithm MAX_TRIES times then we have selected an “optimal” attribute subset. We have not tried all combinations of all attributes as this is intractable for a large data set. However, we have made a sufficient approximation.

Conclusion

In this paper we have introduced a binary neural decision table classifier. The AURA neural architecture, which underpins the classifier, has demonstrated superior training and recall speed compared to conventional indexing approaches such as hashing or inverted file lists which may be used for a decision table. AURA trains 20 times faster than an inverted file list and 16 times faster than a hashing algorithm. It is up to 24 times faster than the inverted file list for recall and up to 14 times faster than the hashing algorithm. In this paper, we described the implementation details of the technique. Our next step is to evaluate the AURA decision table for speed and memory usage against a conventional decision table implementation.

We have shown how two quite different attribute selection approaches may be implemented within the AURA decision table framework. We described a mutual information attribute selector that examines attributes on an individual basis and scores them according to their class discrimination ability. We also demonstrated a probabilistic Las Vegas algorithm which uses random search and inconsistency to evaluate attribute subsets.

We feel the technique is flexible and easily extended to other attribute selection algorithms.

Acknowledgement

This work was supported by EPSRC Grant number GR/R55101/01.

References

- [AA68] Aleksander, I., & Albrow, R.C. **Pattern recognition with Adaptive Logic Elements**. *IEE Conference on Pattern Recognition*, pp 68-74, 1968.
- [ATB84] Aleksander, I., Thomas, W.V., & Bowden, P.A. **Wisard: A radical step forward in image recognition**. *Sensor Review*, pp 120-124, 1984.
- [AA98] Alwis, S., & Austin, J. **A Novel Architecture for Trademark Image Retrieval Systems**. In, *Electronic Workshops in Computing*, 1998.
- [A95] Austin, J. **Distributed Associative Memories for High Speed Symbolic Reasoning**. In, *IJCAI Working Notes of Workshop on Connectionist-Symbolic Integration: From Unified to Hybrid Approaches*, pp 87-93, 1995.
- [A98] Austin, J. **RAM-based Neural Networks**, *Progress in Neural Processing 9*, Singapore: World Scientific Pub. Co., 1998.
- [AKL95] Austin, J., Kennedy, J., & Lees, K. **A Neural Architecture for Fast Rule Matching**. In, *Artificial Neural Networks and Expert Systems Conference (ANNES'95)*, Dunedin, New Zealand, 1995.
- [BB59] Bledsoe, W.W., & Browning, I. **Pattern recognition and Reading by Machine**. In, *Proceedings of Eastern Joint Computer Conference*, pp 225-231, 1959.
- [H01] Hodge, V., **Integrating Information Retrieval & Neural Networks**, *PhD Thesis*, Department of Computer Science, The University of York, 2001.
- [HA01] Hodge, V. & Austin, J. **An Evaluation of Standard Retrieval Algorithms and a Binary Neural Approach**. *Neural Networks* 14(3), pp. 287-303, Elsevier, 2001.
- [HA04] Hodge, V. & Austin, J. **A High Performance k-NN Approach Using Binary Neural Networks**. To appear, *Neural Networks*, Elsevier, 2004.
- [K95] Kohavi, R.. **The Power of Decision Tables**. In, **Procs of European Conference on Machine Learning**. *LNAI 914*, Springer-Verlag, pp174-189, 1995.
- [LS96] Liu, H., and Setiono, R. **A probabilistic approach to feature selection - A filter solution**. In, *13th International Conference on Machine Learning (ICML'96)*, pp. 319-327, 1996.
- [TA00] Turner, A., & Austin, J. **Performance Evaluation of a fast Chemical Structure Matching Method using Distributed Neural Relaxation**. In, *4th International conference on Knowledge Based Intelligent Engineering Systems*, 2000.
- [W94] Wettscherek, D.. **A Study of Distance-Based Machine Learning Algorithms**. *PhD Thesis*, Dept of Comp. Sci., Oregon State University, 1994.
- [ZAK99] Zhou, P., Austin, J. & Kennedy, J. **A High Performance k-NN Classifier Using a Binary Correlation Matrix Memory**, *Advances in Neural Information Processing Systems*, Vol. 11, 1999.